### Monday, October 22, 2018

**12:00pm-12:50pm**  
**Mathematical Biology** -- Padmini Rangamani (UC San Diego, Mechanical and Aerospace Engineering)  
*Geometric Principles of Spatio-temporal Dynamics of Second Messengers in Dendritic Spines* -- 335 West Hall

**3:00pm-4:00pm**  
**Student Dynamics** -- Jasmine Powell (University of Michigan)  
*The mapping class group of a rational map* -- 1060 East Hall

**4:00pm-5:00pm**  
**Complex Analysis, Dynamics and Geometry** -- Yusheng Luo (Harvard)  
*Barycentric extensions* -- 3088 East Hall

**4:00pm-5:20pm**  
**Group, Lie and Number Theory** -- Tony Feng (Stanford)  
*The Artin-Tate pairing on the Brauer group of a surface* -- 4088 East Hall

**3:00pm-6:00pm**  
**Geometry & Physics** -- Aaron Pixton (MIT)  
*Gromov-Witten theory of elliptic fibrations* -- 4096 East Hall

**4:00pm-5:00pm**  
**Student Combinatorics** -- Harry Richman (University of Michigan)  
*Electrifying random trees II: edge correlations* -- 3866 East Hall

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### Tuesday, October 23, 2018

**11:30am-1:00pm**  
**IBL Workshops/Lectures** --  
**IBL Lunch** -- 4866 East Hall

**3:00pm-4:00pm**  
**Student Geometry/Topology** -- Konstantinos Tsouvalas (University of Michigan)  
*Properties T and FA* -- 1866 East Hall

**3:00pm-3:50pm**  
**Student Commutative Algebra** -- Monica Lewis (University of Michigan)  
*The Absolute Integral Closure* -- 4088 East Hall

**4:00pm-5:00pm**  
**Colloquium Series** -- Josselin Garnier (Ecole Polytechnique France)  
*Multiscale analysis of wave propagation and imaging in random media* -- 1360 East Hall

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### Wednesday, October 24, 2018

**3:00pm-3:50pm**  
**Student Arithmetic** -- David Schwein (University of Michigan)  
*Introduction to the Langlands Program* -- 4096 East Hall

**4:00pm-5:00pm**  
**Financial/Actuarial Mathematics** -- Martin Larsson (ETH)  
*Short- and long-term relative arbitrage in stochastic portfolio theory* -- 1360 East Hall

**4:00pm-5:20pm**  
**Algebraic Geometry** -- Bill Fulton (University of Michigan)  
*Determinants and Pfaffians II* -- 4096 East Hall

**4:00pm-5:30pm**  
**Logic** -- Denis Hirschfeldt (Univ. of Chicago)  
*Computability theory, asymptotic density, and algorithmic randomness* -- B735 East Hall

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### Thursday, October 25, 2018

**1:00pm-2:30pm**  
**Student Homotopy Theory** -- Ruian Chen (University of Michigan)  
*What we mean when we talk about... A_infinity and E_infinity* -- 2866 East Hall

**3:00pm-4:00pm**  
**Commutative Algebra** -- Eric Canton (University of Michigan)  
*Log discrepancies and Frobenius splittings* -- 4088 East Hall

**4:00pm-5:00pm**  
**Differential Equations** -- Brian Hall (Notre Dame)  
*A PDE approach to eigenvalue distributions in random matrix theory* -- 4088 East Hall

**4:00pm-5:00pm**  
**Student Algebraic Geometry** -- Haoyang Guo (UM)  
*Riemann-Zariski space and Nagata's compactification* -- 3866 East Hall

**4:00pm-5:00pm**  
**Colloquium Series** -- Denis Hirschfeldt (University of Chicago)  
*Computability and Ramsey Theory* -- 1360 East Hall
### Friday, October 26, 2018

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<tr>
<th>Time</th>
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<tr>
<td>3:00pm-4:00pm</td>
<td><strong>Applied Interdisciplinary Mathematics (AIM)</strong> -- Harbir Antil (George Mason University) <em>Fractional PDEs: control, applications, and beyond</em> -- 1084 East Hall</td>
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<td>3:00pm-4:00pm</td>
<td><strong>Group, Lie and Number Theory</strong> -- Jordan Ellenberg (University of Wisconsin) <em>Heights of rational points on stacks (note special day, time, place)</em> -- 4096 East Hall</td>
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<td><strong>Geometry</strong> -- Pengyu Le (U Michigan) <em>Perturbations of Null Hypersurfaces and the Null Penrose Inequality</em> -- 3866 East Hall</td>
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<td><strong>Combinatorics</strong> -- Zachary Hamaker (University of Michigan) <em>Weak order for monotone triangles</em> -- 2866 East Hall</td>
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<td><strong>Topology</strong> -- Autumn Kent (UWisconsin) <em>Big Torelli groups</em> -- 3866 East Hall</td>
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<tr>
<td>4:00pm-4:50pm</td>
<td><strong>Student AIM Seminar</strong> -- Ryan Kohl (University of Michigan) <em>Fast Multipole Methods</em> -- 1084 East Hall</td>
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<td>5:00pm-6:00pm</td>
<td><strong>Student Machine Learning</strong> -- Julio Cesar Soldevilla Estrada (University of Michigan) <em>Introduction to Graphical Models</em> -- 4088 East Hall</td>
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### Saturday, October 27, 2018

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<tr>
<td>12:00am-12:00am</td>
<td><strong>Special Events</strong> -- (*) <em>Graduate Research Opportunities for Women (GROW)</em> -- 1324 East Hall</td>
</tr>
</tbody>
</table>
Mathematical Biology  
Monday, October 22, 2018, 12:00pm-12:50pm  
335 West Hall  
Padmini Rangamani (UC San Diego, Mechanical and Aerospace Engineering)  
*Geometric Principles of Spatio-temporal Dynamics of Second Messengers in Dendritic Spines*

The ability of the brain to encode and store information depends on the plastic nature of the individual synapses. The increase and decrease in synaptic strength, mediated through the structural plasticity of the spine, are important for learning, memory, and cognitive function. Dendritic spines are small structures that contain the synapse. They come in a variety of shapes (stubby, thin, or mushroom-shaped) and a wide range of sizes that protrude from the dendrite. These spines are the regions where the postsynaptic biochemical machinery responds to the neurotransmitters. Spines are dynamic structures, changing in size, shape, and number during development and aging. While spines and synapses have inspired neuromorphic engineering, the biophysical events underlying synaptic and structural plasticity remain poorly understood.

Our current focus is on understanding the biophysical events underlying structural plasticity. I will discuss two recent efforts from my group - first, a systems biology approach to construct a mathematical model of biochemical signaling and actin-mediated transient spine expansion in response to calcium influx caused by NMDA receptor activation and second, a series of spatial models to study the role of spine geometry and organelle location within the spine for calcium and cyclic AMP signaling. I will conclude with some new efforts in using reconstructions from electron microscopy to inform computational domains. I will conclude with how geometry and mechanics plays an important role in our understanding of fundamental biological phenomena and some general ideas on bio-inspired engineering.

Student Dynamics  
Monday, October 22, 2018, 3:00pm-4:00pm  
1060 East Hall  
Jasmine Powell (University of Michigan)  
*The mapping class group of a rational map*

The iteration of complex functions gives a way of associating to certain rational maps a torus with marked points. In this talk, we see how this association comes about, and we explore the following question: If you continuously vary such a rational map, how does that change the associated torus? No complex dynamics background will be assumed.
We will study the Douady-Earle barycentric extension of maps on $S^{n-1}$. We will show the extension is uniformly Lipschitz if the map is quasi-regular, which, in a sense, is a generalization of Schwarz lemma. Using the bound on the Lipschitz constant, we are able to construct a geometric limit of a degenerating sequence of rational maps on an $\mathbb{R}$-tree. This geometric limit as $\mathbb{R}$-tree is a generalization of Ribbon $\mathbb{R}$-tree construction for compactification of Blaschke products by Curt McMullen, and as an analogue of the Thurston compactification for the surface group studied by Morgan-Shallen, Bestivina, Paulin and Otal.

There is a canonical pairing on the Brauer group of a surface over a finite field, and an old conjecture of Tate predicts that this pairing is alternating. In this talk I will present a resolution to Tate's conjecture. The key new ingredient is a circle of ideas originating in algebraic topology, centered around the Steenrod operations. The talk will advertise these new tools (while assuming minimal background in algebraic topology).

Let $X$ be a variety. Gromov-Witten invariants of $X$ are defined by integrals on the moduli space of stable maps from a genus $g$ curve to $X$. I will describe a conjecture interpreting these invariants as coefficients of certain automorphic forms if $X$ is the total space of an elliptic fibration $X \to B$. I will also discuss a relationship between this conjecture and Givental's R-matrix action on cohomological field theories. This is joint work with Georg Oberdieck.
Student Combinatorics
Monday, October 22, 2018, 4:00pm-5:00pm
3866 East Hall
Harry Richman (University of Michigan)
Electrifying random trees II: edge correlations

Last time Will explained to us how random spanning trees of a given graph are related to electrical networks, and gave a method for efficiently generating random spanning trees. In this talk, we will explore a second application of electrical network theory to random spanning trees by addressing the following question: given two edges in a graph, how does the presence / absence of the first edge affect the presence / absence of the second edge in a random spanning tree?

IBL Workshops/Lectures
Tuesday, October 23, 2018, 11:30am-1:00pm
4866 East Hall

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IBL Lunch

IBL lunch from 11:30am to 1:00pm. It will take place in the faculty lounge (4866 EH), and you should feel free to come for any length of time. Lunch will be provided.

Student Geometry/Topology
Tuesday, October 23, 2018, 3:00pm-4:00pm
1866 East Hall
Konstantinos Tsouvalas (University of Michigan)
Properties T and FA

Property T for locally compact groups was first introduced by Kazhdan in 1967. Examples of groups satisfying Property T include real simple Lie groups of higher rank and their lattices. Another property satisfied by lattices in higher rank simple Lie groups is property FA. In this talk we will see the definition of both properties, discuss the relation among them and give several (non)examples.
Student Commutative Algebra
Tuesday, October 23, 2018, 3:00pm-3:50pm
4088 East Hall
Monica Lewis (University of Michigan)
The Absolute Integral Closure

We'll revisit the familiar notions of module-finite and integral extensions of a domain, and investigate one way in which these notions enable us to talk about "almost" belonging to an ideal. We'll define a kind of maximal integral extension of a domain, analogous to the algebraic closure of a field, that in some ways conceptually simplifies this picture. We'll discuss (without proof) how this construction illustrates a fundamental difference between characteristic p and (equal) characteristic 0.

Colloquium Series
Tuesday, October 23, 2018, 4:00pm-5:00pm
1360 East Hall
Josselin Garnier (Ecole Polytechnique France)
Multiscale analysis of wave propagation and imaging in random media

Wave propagation in random media can be studied by multiscale and stochastic analysis. We review some recent advances and their applications. In particular, in a physically relevant regime of separation of scales, wave propagation is governed by a Schrodinger-type equation driven by a Brownian field. We study the associated moment equations and describe the propagation of coherent and incoherent waves. These results make it possible to introduce and characterize correlation-based imaging techniques. We show how these techniques can be used in random media and/or with incoherent, opportunistic, or ambient noise illumination.

Student Arithmetic
Wednesday, October 24, 2018, 3:00pm-3:50pm
4096 East Hall
David Schwein (University of Michigan)
Introduction to the Langlands Program

The Langlands Program conjectures a relationship between seemingly unrelated classes of mathematical objects, one algebraic and the other analytic: Galois groups of number fields on the one hand, and modular (or more generally, automorphic) forms on the other. The relationship consists of a dictionary between Galois representations and automorphic representations. Most mysteriously (and usefully!) of all, the dictionary is expected to be compatible with the formation of L-functions. In this talk we'll give a modest snapshot of this exciting area of number theory.
Financial/Actuarial Mathematics  
Wednesday, October 24, 2018, 4:00pm-5:00pm  
1360 East Hall  
Martin Larsson (ETH)  
*Short- and long-term relative arbitrage in stochastic portfolio theory*

Stochastic Portfolio Theory is a mathematical framework for studying large equity markets, especially the performance of long-term investments. An important focus are universal features that only depend weakly on specific modeling assumptions. A basic result of this kind states that a mild nondegeneracy condition suffices to guarantee long-term relative arbitrage, that is, the possibility to outperform the market over sufficiently long time horizons. A longstanding open question has been whether short-term relative arbitrage is also implied. A qualitative answer, in the negative, was recently given by Fernholz, Karatzas & Ruf. In this work we settle the question by characterizing and explicitly computing the critical time horizon beyond which relative arbitrage always exists. The key tool is a previously unknown connection between existence of relative arbitrage and certain geometric PDE describing mean curvature flow.

Algebraic Geometry  
Wednesday, October 24, 2018, 4:00pm-5:20pm  
4096 East Hall  
Bill Fulton (University of Michigan)  
*Determinants and Pfaffians II*

The formula for the cohomology class of the locus where a map between vector bundles has at most a given rank was developed from 1849 to 1974. It is a determinant of a matrix of polynomials in Chern classes of the two bundles, given by Giambelli in 1904. The study of symmetric and skew-symmetric bundle maps can be traced back to 1862, but for these with quite a variety of different formulas have been found. Only recently has it become clear that formulas for these loci are best given by pfaffians. These two talks will sketch some of this history, and describe some of the interesting algebra and combinatorics involved in the formulas. As time permits, we'll discuss how these ideas can be used to construct Schubert polynomials in types A, B, C, and D.
Logic
Wednesday, October 24, 2018, 4:00pm-5:30pm
B735 East Hall
Denis Hirschfeldt (Univ. of Chicago)
Computability theory, asymptotic density, and algorithmic randomness

The notion of generic complexity was introduced by Kapovich, Myasnikov, Schupp, and Shpilrain to study problems with high worst-case complexity that are nevertheless easy to solve in most instances. They also introduced the notion of generic computability, which captures the idea of having a partial algorithm that halts for almost all inputs, and correctly computes a decision problem whenever it halts. Jockusch and Schupp began the general computability-theoretic investigation of generic computability and also defined the notion of coarse computability, which captures the idea of having a total algorithm that might make mistakes, but correctly decides the given problem for almost all inputs (although this notion had been studied earlier in Terwijn's dissertation). Two related notions, which allow for both failures to answer and mistakes, have been studied by Astor, Hirschfeldt, and Jockusch (although one of them had been considered in the 1970's by Meyer and by Lynch). All of these notions lead to notions of reducibility and associated degree structures. I will discuss recent and ongoing work in the study of these reducibilities, and in particular connections with the theory of algorithmic randomness.

Student Homotopy Theory
Thursday, October 25, 2018, 1:00pm-2:30pm
2866 East Hall
Ruian Chen (University of Michigan)
What we mean when we talk about... A_infinity and E_infinity

Homotopy theorists often jabber away phrases like "up to all higher homotopies", or "up to a contractible space of choices"; but what do they really mean? In this talk, our only goal is to make sense of these phrases in a precise manner. Specifically, we make use of May's notion of *operad* as a device to encode compositions of abstract operations and their unitality, associativity and commutativity information. Objects with actions of operads, which is to say algebras over operads, can be used to model various notions of "monoid-like" algebraic structures with different levels of associativity and commutativity, from as strict as the ordinary commutative monoids to as weak as the H-spaces. In particular, we will characterize the operads that keep the perfect balance between the strictness and laxness; their algebras, called the A_infinity algebras and E_infinity algebras, then capture the correct notions of "homotopy coherent (commutative) monoids".

The only prerequisite for this talk is the willingness to think about the homotopies involved in higher structures.
Commutative Algebra  
**Thursday, October 25, 2018, 3:00pm-4:00pm**  
**4088 East Hall**  
**Eric Canton (University of Michigan)**  
*Log discrepancies and Frobenius splittings*

Hara and Watanabe proved, roughly, that the Frobenius splitting behavior of sufficiently nice local rings of positive characteristic influence numerical invariants, called log discrepancies, associated to discrete valuation rings dominating such local rings. In this talk, I will sketch and extend their ideas, interpreting their method with the modern lenses of Cartier algebras and Berkovich spaces.

Differential Equations  
**Thursday, October 25, 2018, 4:00pm-5:00pm**  
**4088 East Hall**  
**Brian Hall (Notre Dame)**  
*A PDE approach to eigenvalue distributions in random matrix theory*

One of the lessons of random matrix theory is that the distribution of eigenvalues of a randomly chosen N x N matrix tends to simplify in the limit as N tends to infinity. Consider, for example, N x N matrices chosen randomly with independent, normally distributed entries. In the limit as N tends to infinity, the eigenvalues with high probability become uniformly distributed on a disk of radius 1. This is the "circular law."

In joint work with Bruce Driver and Todd Kemp, I have been studying the eigenvalues of Brownian motion in the general linear group, which is a sort of deformation of the circular law. The limiting eigenvalue distribution in this case is described by a certain first-order nonlinear PDE. The PDE can then be analyzed by the Hamilton-Jacobi method, which involves trying to solve an associated system of ODEs. In the end it turns out that the limiting eigenvalue distribution has a very nice structure.

No prior knowledge of random matrix theory will be assumed and there will be lots of pictures!
Student Algebraic Geometry  
Thursday, October 25, 2018, 4:00pm-5:00pm  
3866 East Hall  
Haoyang Guo (UM)  
*Riemann-Zariski space and Nagata's compactification*

In the first half of the 20th century, Zariski introduced the Riemann manifold $RZ_K(k)$ for a field extension $K/k$, defined as the limit of all proper $k$-models of $K$. This turns out to be a very useful tool, and it is one of the main ingredients in his desingularization for surfaces and threefolds in characteristic 0.

In this talk, we will generalize Zariski's definition and introduce the relative Riemann-Zariski space for any morphism of schemes, following Temkin. Then we give an explicit description of Riemann-Zariski space, using valuation theory. As an application, we will use this to prove the Nagata's compactification in algebraic geometry, saying that given a finite type and separated morphism between two schemes, it can be factored as an open immersion followed by a proper morphism.

Colloquium Series  
Thursday, October 25, 2018, 4:00pm-5:00pm  
1360 East Hall  
Denis Hirschfeldt (University of Chicago)  
*Computability and Ramsey Theory*

Computability theory can be seen as the study of the fine structure of definability. Much of its power relies on the deep connections between definability and computation. These connections can be seen in fundamental results such as Post's Theorem, which establishes a connection between the complexity of formulas needed to define a given set of natural numbers and its computability-theoretic strength. As has become increasingly clear, they can also be seen in the computability-theoretic analysis of objects whose definitions come from notions that arise naturally in combinatorics. The heuristic here is that computability-theoretically natural notions tend to be combinatorially natural, and vice-versa. I will discuss some results and open questions in the computability-theoretic analysis of combinatorial principles, in particular Ramsey-theoretic ones such as versions of Ramsey's Theorem for colorings of countably infinite sets, and versions of Hindman's Theorem, which states that for every coloring of the natural numbers with finitely many colors, there is an infinite set of numbers such that all nonempty sums of distinct elements of this set have the same color.
Applied Interdisciplinary Mathematics (AIM)
Friday, October 26, 2018, 3:00pm-4:00pm
1084 East Hall
Harbir Antil (George Mason University)
Fractional PDEs: control, applications, and beyond

Fractional calculus and its application to anomalous transport has recently received a tremendous amount of attention. In these studies, the anomalous transport (of charge, tracers, fluid, etc.) is presumed attributable to long-range correlations of material properties within an inherently complex, and in some cases self-similar, conducting medium. Rather than considering an exquisitely discretized (and computationally explosive) representation of the medium, the complex and spatially correlated heterogeneity is represented through reformulation of the PDE governing the relevant transport physics such that its coefficients are, instead, smooth but paired with fractional-order space derivatives.

This talk will give an introduction to fractional diffusion. We will describe how to incorporate nonhomogeneous boundary conditions in fractional PDEs. We will cover from linear to quasilinear fractional PDEs. New notions of optimal control and optimization under uncertainty will be presented. We will conclude the talk with an approach that allows the fractional exponent to be spatially dependent. This has enabled us to define novel Sobolev spaces and their trace spaces. Several applications in: imaging science, quantum random walks, geophysics, and manifold learning (data analysis) will be discussed.

Group, Lie and Number Theory
Friday, October 26, 2018, 3:00pm-4:00pm
4096 East Hall
Jordan Ellenberg (University of Wisconsin)
Heights of rational points on stacks (note special day, time, place)

We present a definition of the height of a rational point on an algebraic stack over a global field, which is joint work with Matt Satriano and David Zureick-Brown. This notion allows us, for instance, to unify questions about counting number fields of bounded discriminant, counting rational points of bounded height on varieties, and counting arithmetic objects whose moduli spaces aren't representable as schemes. I will also discuss how this language can be used to make a hopeful remark of Robert Harron and Andrew Snowden into a provable mathematical assertion.
Geometry
Friday, October 26, 2018, 3:00pm-3:50pm
3866 East Hall
Pengyu Le (U Michigan)
Perturbations of Null Hypersurfaces and the Null Penrose Inequality

The Penrose inequality in general relativity is a conjectured inequality between the area of the horizon and the mass of a black-hole spacetime. The null Penrose inequality is the case where it concerns the area of the horizon and the Bondi mass at null infinity along a null hypersurface. An effective method to prove Penrose-type inequalities is to exploit the monotonicity of the Hawking mass along certain foliations. In his thesis [S], Sauter constructed the constant mass aspect function foliation aiming to prove the null Penrose inequality. The behavior of the foliation at past null infinity is an obstacle for his method. An idea to overcome this difficulty, which is suggested in the end of [S], is to vary the null hypersurface to achieve the desired behavior of the foliation at null infinity, leading to a spacetime version of the Penrose inequality. To formalise this idea, one need to study perturbations of null hypersurfaces. I will talk about my work on the study of perturbations of null hypersurfaces and its application to the null Penrose inequality.

References
[S] Sauter, J. Foliations of Null Hypersurfaces and the Penrose Inequality, Diss. ETH No.17842.

Combinatorics
Friday, October 26, 2018, 3:00pm-4:00pm
2866 East Hall
Zachary Hamaker (University of Michigan)
Weak order for monotone triangles

Monotone triangles are a rich extension of permutations that biject with alternating sign matrices. We generalize the notions of weak order and descent sets for permutations to monotone triangles, and show they enjoy many analogous properties. In particular, any linear extension of the weak order gives rise to a shelling order on a poset, recently introduced by Terwilliger, whose maximal chains biject with monotone triangles; among these shellings are a family of EL-shellings.

A key tool in our arguments is an action of the 0-Hecke monoid of type A on monotone triangles, generalizing the usual bubble-sorting action on permutations. This leads to a notion of descent set for monotone triangles. We will discuss some properties of descent sets and other consequences of this action, as well as the possibility of extending it to other types.

This work is joint with Vic Reiner.
Topology
Friday, October 26, 2018, 4:00pm-5:00pm
3866 East Hall
Autumn Kent (UWisconsin)
Big Torelli groups

I will discuss some joint work with Aramayona, Ghaswala, McLeay, Tao, and Winarski on the Torelli subgroups of big mapping class groups.

Student AIM Seminar
Friday, October 26, 2018, 4:00pm-4:50pm
1084 East Hall
Ryan Kohl (University of Michigan)
Fast Multipole Methods

Suppose you know the position and momentum of n stars and you want to compute their trajectories. Newton's Law of gravitation says that each star will impart a force on every other star. There are a lot of stars in the sky, so this sounds like bad news. Fortunately we have the Fast Multipole Method (FMM) to help us compute these interactions in O(n) time. The FMM is a relatively young algorithm with lots of areas of active research. In this talk we'll take a look at the principles that make this amazing algorithm work, as well as applications to solving certain types of elliptic PDEs.

Student Machine Learning
Friday, October 26, 2018, 5:00pm-6:00pm
4088 East Hall
Julio Cesar Soldevilla Estrada (University of Michigan)
Introduction to Graphical Models

I will give a brief introduction to graphical models theory. I will focus on bayesian networks, conditional independence and, time permitting, I will talk about making inference in such models. I will try to mention examples of how these models are used in other areas like image processing or speech processing.

Special Events
Saturday, October 27, 2018, 12:00am-12:00am
1324 East Hall

Graduate Research Opportunities for Women (GROW)

A conference aimed at female-identified undergraduate students who may be interesting in pursuing a graduate degree in mathematics. October 26-28, 2018.

http://www-personal.umich.edu/~kochsc/grow2018.html