Pattern Formation and Partial Differential Equations

Tuesday, September 18 • 4:10 p.m. • Room 1360 East Hall

In three specific examples, we shall demonstrate how the theory of partial differential equations (PDEs) relates to pattern formation in nature: Spinodal decomposition and the Cahn-Hilliard equation, Rayleigh-Bénard convection and the Boussinesq approximation, rough crystal growth and the Kuramoto-Sivashinsky equation. These examples from different applications have in common that only a few physical mechanisms, which are modeled by simple-looking evolutionary PDEs, lead to complex patterns. These mechanisms will be explained, numerical simulation shall serve as a visual experiment. Numerical simulations also reveal that generic solutions of these deterministic equations have stationary or self-similar statistics that are independent of the system size and of the details of the initial data. We show how PDE methods, i.e., a priori estimates, can be used to understand some aspects of this universal behavior. In case of the Cahn-Hilliard equation, the method makes use of its gradient flow structure and a property of the energy landscape. In case of the Boussinesq equation, a “driven gradient flow”, the background field method is used. In case of the Kuramoto-Sivashinsky equation, that mixes conservative and dissipative dynamics, the method relies on a new result on Burgers' equation.

Optimal Estimates in Stochastic Homogenization

Wednesday, September 19 • 3:10 p.m. • Room 4088 East Hall

In many applications, one has to solve an elliptic equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in situations where the coefficients are characterized in statistical terms: their statistics are assumed to be translation invariant and to decorrelate over large distances. In this situation, the solution operator behaves—on large scales—like the solution operator of an elliptic problem with homogeneous, deterministic coefficients! Hence the relation between the statistics of the coefficients and the value of the homogenized coefficients is of practical importance. Theory provides a formula for the homogenized coefficients, based on the construction of a “corrector”, which defines harmonic coordinates. This formula has to be approximated in practice. In this talk, we give sharp estimates on the corrector that allows us to assess these approximations. These estimates use tools from statistical mechanics and elliptic regularity theory. This is joint work with A. Gloria and S. Neukamm.

Domain Patterns in Thin-Film Ferromagnets

Thursday, September 20 • 4:10 p.m. • Room 4088 East Hall

From the point of view of mathematics, micromagnetics is an ideal test-bed for a pattern-forming system in materials science: There are abundant experiments on a wealth of visually attractive phenomena and there is a well-accepted continuum model. In this talk, I will focus on a couple of specific experimental domain patterns for thin film ferromagnetic elements. Starting point for our analysis is the micromagnetic model which has several characteristic length scales and thus many parameter regimes. For the pattern under consideration, we identify the appropriate parameter regime and rigorously derive a reduced model. We analytically investigate and numerically simulate the reduced model and compare its predictions to experimental data. Through the analysis of local minimizers and their instabilities we gain insight into the origins of hysteresis.

A reception for Professor Otto will be held at 5:00 p.m.
Tuesday, September 18, in the Mathematics Upper Atrium, East Hall

The Ziwet Lectures were established in 1934 through a bequest from Professor Alexander Ziwet, a faculty member and Chair of the UM Department of Mathematics from 1888-1925. He stipulated that his estate “should be used for the promotion of scientific work.” The Ziwet lectures are one of the most prestigious lectures series in the Department.