Robust and high order marching schemes for the wave equation

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We seek to construct and implement robust, stable, high order, and explicit marching schemes for the wave equation in complex domains using an embedded uniform Cartesian grid. Cartesian grids have advantages over body fitted grids in that they allow for greater speed and easier implementation of numerical algorithms for problems in irregular domains, especially in three dimensions.

In an irregular domain with an embedded uniform Cartesian grid, there will be small cells where the grid intersects the boundary. A standard finite difference marching scheme is restricted in the size of the time step it can take, which is usually dependent on the size of the smallest cell in the spatial discretization. We would like to overcome this time step restriction and construct schemes which are insensitive to the presence of small cells.

In [1] a class of explicit marching schemes was proposed to solve the wave equation in the domain $\mathcal{D} \in \mathbb{R}^d$, $d = 1, 2$, or 3. They are of the form,

$$\hat{u}(x_i, t_{n+1}) = 2\tilde{u}(x_i, t_n) - \tilde{u}(x_i, t_{n-1}) + \int_{y \in \overline{B}(x_i, dt) \bigcap \mathcal{D}} G_d(|y - x_i|, dt)\, \Delta \tilde{u}(y, t_n)\, dy, \tag{1}$$

where $\overline{B}(x_i, dt)$ denotes the closed ball in $\mathbb{R}^d$ of radius dt centered at x_i, and where $G_d(r, dt)$ is the kernel in dimension d. For a point x_i which is within dt of the boundary of $\mathcal{D}$, a correction, using information on the boundary, is made to $\hat{u}(x_i, t_{n+1})$ in order to obtain $\tilde{u}(x_i, t_{n+1})$. Otherwise $\tilde{u}(x_i, t_{n+1}) = \hat{u}(x_i, t_{n+1})$.

We have implemented these schemes in one and two dimensions to high order. In this talk I will discuss interpreting the integral in (1) in such a way as to account for possible discontinuities in the approximating function $\tilde{u}$. I will also give analytical and numerical results relating to the convergence and stability of these schemes, and describe the implementation of the boundary correction in two dimensions.

This is joint work with Leslie Greengard.

References

[1] B. Alpert, L. Greengard, and T. Hagstrom, *An integral evolution formula for the wave equation*, J. Comput. Phys., 162 (2000), pp. 536–543.