A Boundary Integral/Treecode Approach for Plasma Simulations

Dr. Andrew J. Christlieb

Nov. 12^{th}, 2004

Many problems in plasma physics are modeled using a Lagrangian approach. One of the most common models is Particle-In-Cell (PIC), where the system is modeled as a collection of macro particles which interact through long range forces. To compute long range forces in under $O(N^2)$ operations, the particles are interpolated to a mesh where the equations are solved. Afterwards, the fields are interpolated back to the macro particles. The major problems with this approach are that it: can not resolve step plasma gradients, has cumulative interpolation errors, and has difficulty describing arbitrary domains.

The current work seeks to overcome these limitations in particle plasma models by developing a grid-free approach to plasma simulations. The Boundary Integral/Treecode (BIT) method merges boundary integral formulations with treecode algorithms. Using Green's theorem, and setting the functions in Green's theorem to $u = G$ and $v = \Phi$, we can express Poisson's equation as an integral equation

$$\Phi(\mathbf{y}) = \int\int_{\Omega} \frac{\rho(\mathbf{x})}{\epsilon_0} G(\mathbf{x}|\mathbf{y}) d\Omega - \oint_{\partial\Omega} (\Phi(\mathbf{x}) \nabla_{\mathbf{x}} G(\mathbf{x}|\mathbf{y}) - G(\mathbf{x}|\mathbf{y}) \nabla \Phi(\mathbf{x})) \cdot \mathbf{n} ds \ , \qquad (1)$$

where $G(\mathbf{x}|\mathbf{y})$ is the free space Green's function for the Laplace operator. If we are modeling the system as a collection of point charges, ρ is given by $\rho = \sum_{i=1}^{N} q_i \delta(\mathbf{x} - \mathbf{z}_i)$. The contribution to the field from the charge density reduces to

$$\int\int_{\Omega} \frac{\rho(\mathbf{x})}{\epsilon_0} G(\mathbf{x}|\mathbf{y}) d\Omega = \sum_{i=1}^{N} \frac{q_i G(\mathbf{z}_i|\mathbf{y})}{\epsilon_0} \ . \qquad (2)$$

Choosing an appropriate colocation method for the boundary integral formulation effectively reformulates the boundary as a set of point charges, making the field evaluation mesh-free and capable of handling complicated domains. The operation count in the field evaluation is reduced from $O(N^2)$ to $O(N \log N)$ by using a treecode algorithm, which gains its efficiency in essence by treating clusters of particles at a distance as a point charges at the center of the cluster. (More accurate approximations are used in this work.)

The method has been applied to several bonded plasmas, including the formation of a virtual cathode. When compared with PIC and direct summation, obvious differences are self evident. The direct sum and treecode solutions agree to machine error, while the PIC results have obvious holes that open up in phase space (Figure 1).

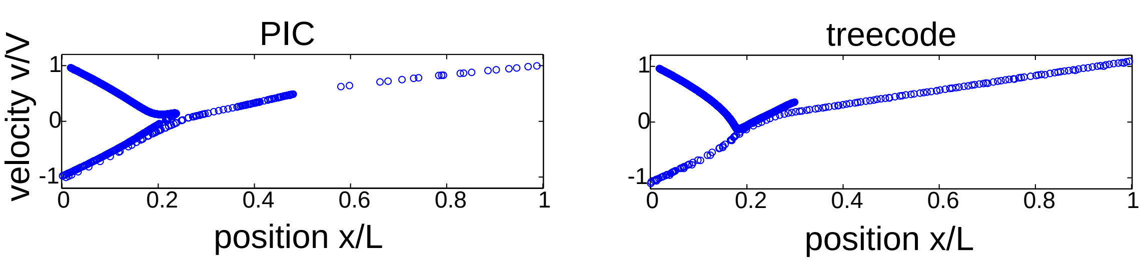

Figure 1: Phase space for a simulations of a 1D virtual cathode. Left hand image, PIC solution. Right hand image, direct sum and treecode solution.