The classical Grothendieck ring over an algebraically closed field is the Abelian group on isomorphism classes of varieties modulo the "scissor" relations $[X] = [Y] + [X - Y]$ for $Y \subset X$, with multiplication given by the Cartesian product of varieties. After Konsevitch introduced in the mid 90s motivic integration—a generalization of p-adic integration taking instead values in the Grothendieck ring—Denef and Loeser expanded his technique using certain Quantifier Elimination results in valued fields, to obtain the "motivic rationality" (=rationality as a series with coefficients in the Grothendieck ring) of several generating series, like the (geometric/motivic) Igusa zeta series. However, their usage of embedded resolution of singularities essentially confines their results to characteristic zero. I will describe a further generalization of their theory, by replacing the classical Grothendieck ring by its "schemic" version, where instead of just varieties, we allow also (non-reduced) quasi-projective schemes. Obviously, the complement of a scheme no longer makes sense, and so we have to put this in an even larger context. Model-theorists have known, especially since Denef and Loeser's work, how to construct a Grothendieck ring of an arbitrary first-order theory, using formulae instead of the sets they define. This latter shift of perspective can also be found in algebraic geometry, where one may view a scheme as the functor it represents. I will explain how both points of view can be reconciled, by equating schemes with schemic, and more generally, positive primitive formulae, and equating formal schemes with formularies, an infinitary, non-first order version of a formula. The latter will serve, in a sense to be explained, as complements. For instance, in the schemic Grothendieck ring (where I denoted a scheme by its coordinate ring, we have the cute formula $k[x] = k[x, 1/x] + k[[x]]$, where I denoted, for emphasis, a scheme by its coordinate ring.

I am aware that the usage of logical jargon (which I promise to keep to a minimum in my talk) might put off the average algebraist/geometer, but in its defense, the theory, apart from being more elementary, has a much more functorial behavior. For instance, it allows for a substitute of motivic integration via the notion of an arc scheme (generalizing the classical notion of a truncated arc space). As a result, I was able to prove motivic rationality for certain hypersurfaces, in any characteristic. To give just one example, albeit in low dimension: every canonical surface singularity (=Du Val surface) admits a rational geometric Igusa zeta series. A further advantage of this theory is that we can now also formulate and study other motivic versions of generating series, like the Hilbert series, which make no sense in the classical setup.