Coarse Schottky problem and Equivariant cell decomposition of Teichmuller spaces

Lizhen Ji - Michigan

In this talk, I will explain some similar results and interaction between locally symmetric spaces and moduli spaces of curves.

First, let A_g be the moduli space of principally polarized abelian varieties of dimension g, the quotient of the Siegel upper space by $Sp(2g, Z)$, and M_g be the moduli space of projective curves of genus g. Then there is a Jacobian map $J : M_g \to A_g$, by associating to each curve its Jacobian.

The celebrated Schottky problem is to characterize the image $J(M_g)$. Buser and Sarnak viewed A_g as a complete metric space and showed that $J(M_g)$ lies in a very small neighborhood of the boundary of A_g as g goes to infinity. Motivated by this, Farb formulated the coarse Schottky problem: determine the image of $J(M_g)$ in the asymptotic cone (or tangent space at infinity) $C_\infty(A_g)$ of A_g, as defined by Gromov in large scale geometry. In a joint work with Enrico Leuzinger, we showed that $J(M_g)$ is d-dense in A_g for some constant d and hence its image in the asymptotic cone $C_\infty(A_g)$ is equal to the whole cone.

Another example is that the symmetric space $SL(n, R)/SO(n)$ admits several important equivariant cell decompositions with respect to the arithmetic group $SL(n, Z)$ and hence a cell decomposition of the locally symmetric space $SL(n, Z) \setminus SL(n, R)/SO(n)$. One such decomposition comes from the Minkowski reduction of quadratic forms (or marked lattices). We generalize the Minkowski reduction to marked hyperbolic Riemann surfaces and obtain an equivariant weak cell decomposition of the Teichmuller space T_g with respect to the mapping class groups Mod_g.