

**Number theory and Representation
Theory seminar, Fall 2007
East Hall 4096
September 17, 3:10-5:00pm
(tea break at 4pm)**

Bilinear forms invariant under algebraic groups

Skip Garibaldi (Emory)

Abstract

For representations of semisimple Lie algebras over the complex numbers, there is a well-known criterion in terms of highest weights for whether the representation has a symmetric bilinear form that is invariant under the action of the Lie algebra, i.e., whether the representation is orthogonal. If it is, then the bilinear form is uniquely determined up to a scalar. These results are easily extended to irreducible representations of semisimple algebraic groups over arbitrary fields. For non-split groups, the bilinear form gives some information about the group. One famous example is the Killing form for simple Lie algebras over the real numbers.

It turns out that these invariant forms are already interesting for groups of type A_1 .