LECTURES IN CR GEOMETRY

UNIVERSITY OF MICHIGAN, ANN ARBOR

Joël Merker (ENS)

Local CR geometry, CR orbits, Hilbert transform in Hölder spaces and the local Bishop equation. CR manifolds carry a constant rank induced tangential antiholomorphic distribution which, generically, is not Frobenius-integrable. Beyond Chow's or Nagano's Lie bracket spanning condition, Sussmann orbits are the adequate concept in the smooth category. Usually, holomorphic discs attached to CR manifolds are constructed in Hölder spaces, because of their flexibility for norm computations, Picard iteration, and implicit function theorem.

Demailly-Semple jets of orders 4 and 5 in dimension 2. Siu and Demailly's strategy for Kobayashi's hyperbolicity conjecture involves the construction of global holomorphic jet differentials wich are (locally) invariant under reparametrization. For jets of orders 2, 3 and 4 in dimension 2; for jets of of order 2 and 3 in dimension 3, the so-called bracket procedure generates all the invariants (3, 5, 9; 6, 16 respectively). For jets of order 5 in dimension 2, the 36 bracket invariants share 210 syzygies ; for jets of order 6 in dimension 2, there would exist 210 bracket invariants sharing 14 950 syzygies. However, already for jets of order 5, we show that bracketing is not enough, and maybe, infinitely many invariants exist, as in Nagata's counterexample to Hilbert's 14th problem-conjecture. Strikingly, 5 is also the minimal expected degree for Kobayashi-hyperbolicity of surfaces of $\mathbb{P}_3(\mathbb{C})$.

Egmont Porten (Mid Sweden University)

Geometry of CR orbits and holomorphic extension. One of the fundamental questions in the study of CR functions on embedded CR manifolds is whether they extend holomorphically to open wedges in the ambient space. If the answer is positive, one can deduce very strong structural information. We will explain the optimal extension theorems which link holomorphic extension with the structure of CR orbits. The original proofs are due to Trépreau and Tumanov in the local, and to Jöricke and Merker in the global case.

We will explain a recent, relatively simple argument which shows a very transparent link between differential geometry and extension properties and does apply to the local and global settings uniformly.

Levi Flat Fillings of Spheres and the Continuity Principle. The classical continuity principle tells how to use families of holomorphic discs in the geometric study of envelopes of holomorphy. In order to gain flexibility in applications, it is desirable to have more general versions of the continuity principle applying to families of general holomorphic curves. In the talk, we will mainly concentrate on an example illustrating various hidden obstructions to possible generalizations. We will construct a 2-sphere $S \subset \mathbf{C}^2$ bounding an embedded Levi flat 3-ball along which holomorphic extension from neighborhoods of S fails. Interestingly, the same S also bounds an immersed Levi flat ball with good extension properties. This is joint work with Burglind Jöricke, Bonn.