RTG WORKSHOP IN CR GEOMETRY

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Joël Merker (ENS Paris)

Morse-theoretical proof of the Hartogs extension theorem (1906). In 1998, Fornæss showed, by means of a topological example constructed by drilling a snake-like self-biting hole in the unit bidisc of $\mathbb{C}^2$ that the classical Hartogs extension theorem cannot be proved by pushing analytic discs with the constraint that all discs remain inside the domain. Relaxing this constraint and using auxiliary collapsing spheres, we Hartogs-fill any arbitrary domain, controlling the topology thanks to Morse theory.

Removable singularities of codimension one in generic manifolds of CR dimension one. The classical Painlevé problem asks for conditions insuring removability of closed singularities for locally bounded holomorphic functions in the complex plane. Denjoy studied the question for real analytic or smooth arcs. We will survey recent, sharp generalizations to the category of CR manifolds.

Han Peters (University of Wisconsin)

Time Averages for Polynomials. (Joint work with Stefan Maubach.) The only invertible polynomials in the complex plane are the affine functions. However, in higher dimensions there are many interesting invertible polynomial endomorphisms.

The polynomial automorphism group in two complex variables is well understood: every polynomial automorphism is a finite composition of "elementary maps". This result is extremely useful for many applications including dynamical systems.

Recently it was shown that in dimensions 3 and higher the polynomial automorphisms do not generate the polynomial automorphism groups. There are several conjectured generator sets but no useful generator set is known. We wil generalize one of these generator sets called locally finite polynomial mappings by defining polynomials that admit a (local or global) time average.

To obtain a better understanding of this definition we study time averages first for polynomials in the complex plane.

Egmont Porten (Mid Sweden University)

Real discs in nonpseudoconvex boundaries. In the last two decades, removability of singularities of CR functions was a topic of active research, motivated also by strong links to polynomial convexity, envelopes of holomorphy and approximation theory. A spectacular result of Jöricke from the beginnings of the topic tells that totally real discs in strictly pseudoconvex boundaries in $\mathbb{C}^2$ are removable. For a while the role of pseudoconvexity was unclear, until it turned out that it can be completely unneccessary. I will explain a global argument for this relying on filling techniques for 2-spheres. In retrospect, this may be considered as a first step towards a much more general theory of removable singularities of codimension one in generic manifolds of CR dimension one.

The Hartogs extension theorem on complex spaces. The shortest way to prove the classical Hartogs theorem is to deduce it from vanishing theorems for $\bar{\partial}$. The argument can be extended to $(n-1)$-complete complex manifolds, as shown by Andreotti-Hill. The corresponding problem for complex spaces had been open for quite a while, probably because no appropriate extension/substitute for $\bar{\partial}$-techniques was available. We will explain a solution of the problem, which succeeds by combining a Morse theoretical approach to the classical Hartogs theorem with techniques for almost plurisubharmonic functions.

Jean-Pierre Rosay (University of Wisconsin)

Extension of holomorphic bundles. Although I have not made very recent progress on that question, I wish to give some "publicity" to the question of extending holomorphic bundles. My interest in the question came from a counterexample that I gave of a non-Stein bundle over the unit disc with polynomial fiber automorphisms. In particular it came with the completely trivial observation that the famous Skoda bundle extends to a holomorphic bundle over the unit disc (a fact that had been completely overlooked).