WEAK REFLECTION PRINCIPLE FOR DIFFUSIONS, WITH APPLICATIONS IN FINANCE AND PHYSICS

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ABSTRACT. Consider a regular diffusion on a real line, with the transition semigroup \((P_t)\). We find a linear one-to-one mapping \(S\) between the spaces of test functions on the negative and the positive half lines, respectively. This mapping has the following symmetry property with respect to the diffusion semigroup:

\[ P_t S f(0) = P_t f(0), \quad \forall t \geq 0. \]

In the case of a Brownian motion, or any other diffusion that is symmetric with respect to zero, this result follows from the classical reflection principle. However, because of the weak formulation of the above result, we were able to extend it to the class of diffusion processes which do not possess any a priori symmetry properties. We, therefore, call it a Weak Reflection Principle for Diffusions.

The mapping \(S\) is constructed explicitly in a way well-suited for computations.

We provide various applications of the above result, including: computation of the joint distribution of the process and its running maximum, static hedging of barrier options, and offsetting the temperature oscillations at a given point in a heterogeneous material.