CR GEOMETRY AND HERMITIAN ANALOGUES OF HILBERT’S 17-TH PROBLEM

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Abstract: Artin’s solution (1925) of Hilbert’s 17th problem states that a polynomial \( p \) in \( k \) real variables is non-negative if and only if there is a polynomial \( q \) such that \( q^2p \) is a sum of squares. Pfister showed that \( 2^k \) squares suffice. One Hermitian analogue would be to write a non-negative Hermitian polynomial \( f(z, \bar{z}) \) as a (Hermitian) squared norm \( \|h(z)\|^2 \), but doing so is not always possible. We might also try to write a Hermitian polynomial as a squared norm on an algebraic set. We give background, classical examples, a recent necessary condition which requires a matrix version of non-negativity, and if time permits, a simple example showing that the Hermitian analogue of Pfister’s result fails.