Prismatic cohomology of stacks and Totaro’s inequality

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In his recent work “Hodge theory of classifying stacks” Totaro computed the de Rham cohomology of the classifying stack of a split reductive group $G$ over $\mathbb{F}_p$ and showed that it agrees with the $\mathbb{F}_p$-singular cohomology of the classifying space $BG(\mathbb{C})$ under the assumption that $p$ is non-torsion for $G$. He also proved that $\dim H^{32}_{dR}(B\text{Spin}(n)_{\mathbb{F}_2}) > \dim H^{32}(B\text{Spin}(n)(\mathbb{C}), \mathbb{F}_2)$, thus showing that the equality between the dimensions does not hold in general. Nevertheless he suggested that one could probably use $p$-adic Hodge theory to show that at least the "$\geq"$ inequality always holds. I will talk about my work (in progress) with A.Prikhodko, where we show that the theory of prismatic cohomology, used wisely, indeed allows to prove this. If time permits I will also discuss some related questions, like prismatic characteristic classes and an explicit computation of the de Rham cohomology ring of $B\text{Spin}(n)_{\mathbb{F}_2}$ for any $n$. 