Problem Set 4 w. solns

Section 13.6:

1) Use traces to identify and sketch the surface $9x^2 - y^2 + z^2 = 0$.

Solution:
Traces in $x = k$ are $y^2 - z^2 = 9k^2$, family of hyperbolae for $k \neq 0$ and intersecting lines for $k = 0$. Traces in $y = k$ are $9x^2 + z^2 = k^2$, $K \geq 0$, family of ellipses, traces in $z = k$ are $y^2 - 9z^2 = k^2$ again ellipses for $k \neq 0$.

Graph is an elliptic cone with axis the $y$-axis, vertex origin.

2) Consider the equation $y^2 = x^2 + 4z^2 + 4$. Reduce to standard form, classify the surface and sketch.

Solution:
Put in form $-x^2/4 + y^2/4 - z^2 = 1$, hyperboloid of two sheets with axis the $y$-axis.

Section 13.7:
3) Plot the point whose spherical coordinates are \((4, -\pi/4, \pi/3)\). Then find its Cartesian coordinates.

**Solution:**

\[
egin{align*}
x &= \sin \pi/3 \cos(-\pi/4) = 4\sqrt{3}/2\sqrt{2}/2 = \sqrt{6} \\
y &= 4\sin \pi/3 \sin(-\pi/4) = 4\sqrt{3}/2(-\sqrt{2}/2) = -\sqrt{6} \\
z &= 4 \cos \pi/3
\end{align*}
\]

4) A solid lies about the cone \(z = \sqrt{x^2 + y^2}\) and below the sphere \(x^2 + y^2 + z^2 = z\). Write a description of the solid in terms of inequalities involving spherical coordinates.

**Solution** Since solid is above the cone \(z \geq \sqrt{x^2 + y^2}\) or \(z^2 \geq x^2 + y^2\) or \(2z^2 \geq x^2 + y^2 + z^2 = \rho^2\) or \(2\rho^2 \cos^2 \phi \geq \rho^2\). Thus \(\cos \phi \geq 1/\sqrt{2}\) since the cone opens upwards. Thus \(0 \leq \phi \leq \pi/4\).

On the other hand in spherical coords the sphere here is \(\rho \cos \phi = \rho^2\) so \(0 \leq \cos \phi\) since the solid lies below the sphere.

These are the two inequalities.

**Section 14.1:**

5) Sketch the curve \(< \sin \pi t, t, \cos \pi t >\)

**Solution**

Since \(x^2 + y^2 = 1\) the curve lies on this cylinder. The curve is a helix which spiral to the right along the cylinder.
6) Show that the curve with parametric equations \( x = t \cos t, y = t \sin t, z = t^2 > \) lies on the cone \( z = x^2 + y^2 \) and use this fact to help sketch the curve.

**Solution:** Here \( x^2 + y^2 = t^2 = z^2 \) so the curves lies on this cone and is a spiral on this cone.

**Section 14.2:**

7) Let \( r(t) = (t, t^2, t^3) \). Find \( r'(t), T(1), r''(t) \) and \( r'(t) \times r''(t) \).

**Solution** \( r' = <1, 2t, 3t^2> \) \( T = r'(1)/|r(1)| = 1\sqrt{14} <1, 2, 3> \) \( r''(t) = <0, 2, 6t> \) so \( r'(t) \times r''(t) = <6t^2, -6t, 2> \).

8) At what points do the curves \( r(t) = <t, 1-t, 3+t^2> \) and \( r(t) = <3-s, s-2, s^2> \) intersect? Find the angle of intersection.

**Solution**
Solve \( t = s - 3, 1 - t = s - 2, 3 + t^2 = s^2 \). Solving for two of the equations and checking the third gives \( t = 1, s = 2 \).
Thus the point of intersection is \((1, 0, 4)\).

Now compute \(r'\) for the two curves at \(t = 1, s = 2\) resp.

We get \(<1, 1, 2>\) and \(<-1, 1, 4>\) respectively.

Computing the angle between these vectors gives \(\theta\) about 55 degrees.