Math 658. Fall 2016

Problem Set 4

1. Show that the form of problem 7 of Problem Set 3 is symplectic.

2. Complete the derivation discussed in class of the dynamics of a particle in a magnetic field from the free particle Hamiltonian.

3. Show that the Hamiltonian does not provide a Lyapunov function for the stable equilbiria of the rigid body equations. Show that physical dissipation destabilizes the equilibrium corresponding to the smallest moment of inertia.

3. Show that the solutions of Cherry's equations discussed in class satisfy Hamilton's equations and discuss their significance.

5. Find a feedback for the controlled rigid body with torque about the minor axis discussed in class that stabilizes the system about its middle axis.

6. Consider the knife edge equations below. Write down the 2 control vector fields, compute their Lie bracket. Is the system controllable? Find the optimal control equations via Lagrange mutlipliers.

$$\min \int (u^2 + v^2) dt$$

subject to the equations

$$\begin{aligned} \dot{x} &= u \cos\theta, \\ \dot{y} &= u \sin\theta, \\ \dot{\theta} &= v\,. \end{aligned}$$

7. Find the infinitesimal generator of the action of the circle group on $\mathbb{R}^2$ and find the corresponding momentum map.

8. a) Write down the general left invariant fields on the special Euclidean group of the plane.

b) Hence write down the general kinematic control system for a planar body

c) Use this to write down the the control problem for a kinematic car (kinematic knife edge).