

# The flapping states of a flag in an inviscid fluid: bistability and the transition to chaos

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(Dated: August 20, 2008)

PACS numbers: 46.40.Ff, 46.40.Jj, 47.15.Ki, 47.52.+j

In our letter [1], an error was made in the computation of the nondimensional flag mass  $R_1 = \rho_s/\rho_f L$  and dimensionless rigidity  $R_2 = B/\rho_f U^2 L^3$ . Here  $\rho_s$  is flag mass per unit length,  $\rho_f$  is fluid mass per unit area,  $B$  is flag bending rigidity and  $U$  is the fluid stream speed.  $L$  is defined as the flag length, but  $R_1$  and  $R_2$  in [1] were computed using only the half flag length  $L/2$  in place of  $L$ . Therefore the values of  $R_2$  in [1] are a factor of 8 too large (due to the factor of  $L^3$ ), and the values of  $R_1$  are a factor of 2 too large.

Figure 1 is a revised version of Figure 2 in [1] comparing the stability boundary with those of recent studies by [2, 3] and [4] (Fig. 3, dashed-dotted line in that work). The correction moves our boundary considerably closer to the others.

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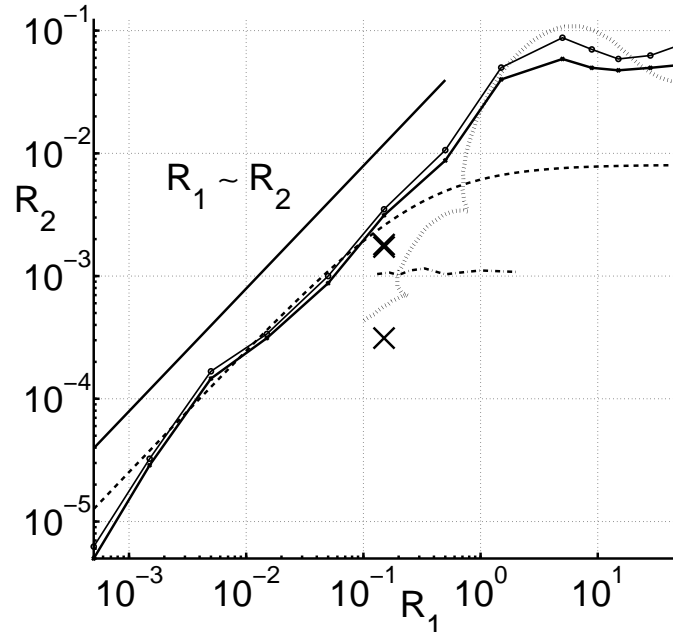


FIG. 1: Computed stability boundary in the  $R_1 R_2$ -plane. The upper solid boundary gives the smallest  $R_2$  above which a small leading-edge forcing ( $y(t) = 10^{-5}(2t)^2 e^{-(2t)^2}$ ) does not lead to flapping. The lower solid boundary is the largest  $R_2$  below which such forcing leads to exponential growth of elastic energy in time until the flag saturates with  $O(1)$  flapping, as shown in Figs. 1 and 2 of [1]. The solid line gives the scaling  $R_1 \sim R_2$  for comparison at small flag masses. The black crosses mark the cases shown in Fig. 2 of [1] (upper cross is (a) and (b), and lower is (c)). The dashed line shows the stability boundary from [2], and the dash-dotted the corresponding boundary plotted in Fig. 3 of [3] (showing only the portion  $R_2 \approx \text{constant}$ , but having the same asymptotic scalings as the model here and in [2]). The dotted line (with cusps) is the 2D result from the model of [4].