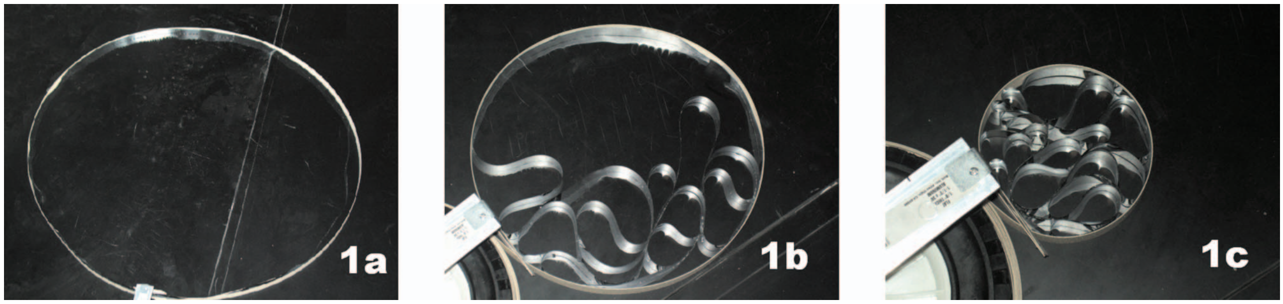

FIG. 1. (Color)

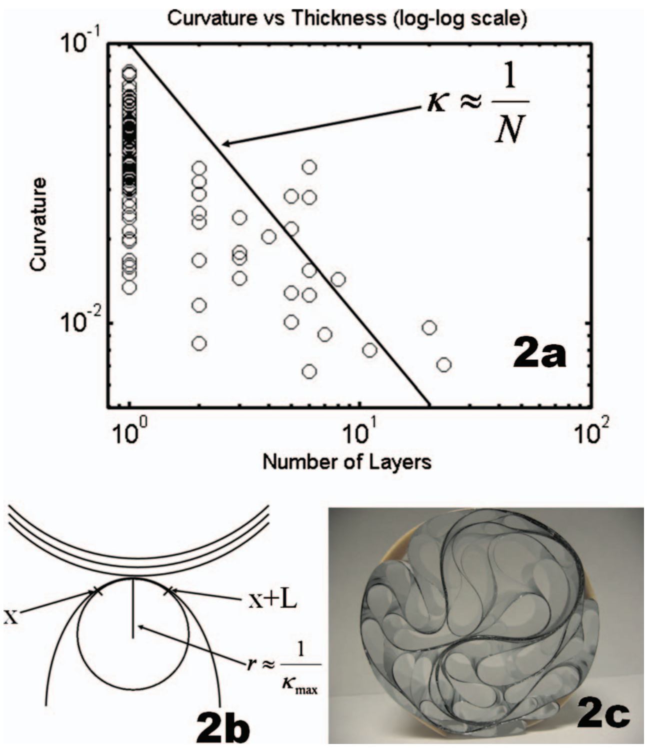

FIG. 2. (Color)

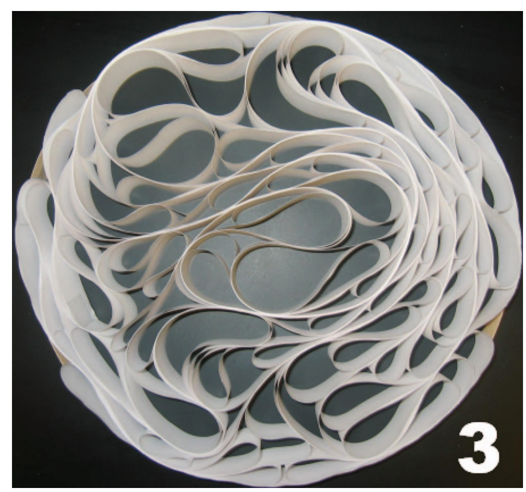

FIG. 3. (Color) Confined ring with paper; length=3048 cm; radius=15 cm.

A cascade of length scales in elastic rings under confinement

Kevin Spears and Silas Alben
Georgia Institute of Technology, Atlanta, Georgia 30332-0160, USA
(Received 17 September 2008; published online 31 December 2008)
[DOI: 10.1063/1.2997338]

Elastic objects under confinement are common in mechanics and biology. Examples include mitochondria and chromosomes, for which conformation and function are strongly determined by confining forces. When elastic objects grow in a confined space, minimization of elastic energy creates a complex spatial configuration and force network. To simulate a two-dimensional ring growing within a rigid circular boundary with a fixed radius, we take a long strip of elastic material (mylar) of fixed length, join the ends to form a closed loop, and then shrink the confining ring boundary (Fig. 1). The loop contacts and overlaps itself in many locations [Figs. 1, 2(c), and 3]. The loop trajectory is discretized and its curvature computed. Where local maxima in curvature occur, the thickness is measured by counting the number of overlapping layers. Figure 2(a) shows a scatter plot of this data for the shape in Fig. 2(c), and we find that the magnitude of a curvature maximum decreases with the number of layers at that point. To quantify this relationship, we consider a region in which two portions of the elastic loop are in contact [Fig. 2(b)]. Because the contact forces balance and are dominated by the curvature maxima, we find also that the elastic energy stored near each maximum is approximately equal. The elastic energy is $\int_x^{x+L} B\kappa(s)^2/2ds$. Here $[x, x+L]$ is the portion of the loop where the curvature is large, B is bending rigidity, and κ is curvature. The curvature near the maximum is of order $\kappa \approx \kappa_{\mathrm{max},N}$, while the length of the region of the large curvature is approximately $1/\kappa_{\mathrm{max},N}$. The integral thus scales as $(B\kappa_{\mathrm{max},N}^2/2)(1/\kappa_{\mathrm{max},N}) = B\kappa_{\mathrm{max},N}/2$. The elastic energy of N layers is $NB\kappa_{\mathrm{max},N}/2$ and, in equilibrium, this term is approximately constant for regions in contact, which implies $\kappa_{\mathrm{max},N} \sim 1/N$.

1054-1500/2008/18(4)/041109/1/$23.00

© 2008 American Institute of Physics

Author complimentary copy. Redistribution subject to AIP license or copyright, see http://cha.aip.org/cha/copyright.jsp