MATH/STATS 425, Homework 10, Do NOT hand in.

Problem 1. Suppose that X and Y are independent identically distributed binomial random variables with parameters (n, p). Calculate the expected value of X given that $X + Y = m$.

Problem 2. Suppose that A and B each randomly, and independently, choose 3 of 10 objects. Find the expected number of objects:

(a) chosen by both A and B;

(b) chosen by exactly one of A and B;

(c) chosen by either A or B;

(Hint: For (a), let X_i be the random variable defined by $X_i = 1$ if the object i is chosen by both A and B, and $X_i = 0$ if not. Then the number of objects chosen by both A and B is $X = X_1 + X_2 + \cdots + X_{10}$.)

Problem 3. A group of n men and n women are lined up at random. Find the expected number of men who have a woman next to them.

(Hint: Consider $2n$ seats. Let X_i be the random variable that is 1 if a man sits at seat i and a woman seats next him (either on the left or on the right).)

Problem 4.

(a) A fair die is thrown. Let A and B be the random variables defined by

$$A = \begin{cases} 1 & \text{the roll is 1} \\ 0 & \text{otherwise} \end{cases}$$

$$B = \begin{cases} 1 & \text{the roll is 2} \\ 0 & \text{otherwise} \end{cases}$$

Compute $Cov(A, B)$.

(b) Let X be the number of 1's and Y be the number of 2's that occur in n rolls of a rfair die. Compute $Cov(X, Y)$.

(Hint: Set $X = X_1 + \cdots + X_n$ where X_i is 1 if ith die shows 1)

Problem 5: The joint density of X and Y is given by:

$$f(x,y) = \begin{cases} \frac{e^{-y}}{y} & 0 < x < y, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Compute $E[X^3|Y = y]$.