MATH 425 Homework 2 (due Friday, February 1)

Problem 1. There are 5 hotels in a certain town. If 3 people check into hotels in a day, what is the probability that they each check into a different hotel?

Problem 2. A pair of fair dice, one of them red and the other blue, is rolled. Calculate the probability that the blue die lands on a higher value than does the red one. (The dice are fair.)

Problem 3. Two dice are thrown n times in succession. Determine the probability that double 2 appears at least once.

Problem 4. An urn contains $n > 0$ red and $m > 0$ black balls.

(a) If two balls are randomly withdrawn, what is the probability that they are the same color?

(b) If a ball is randomly withdrawn and then returned to the urn before the second one is drawn, what is the probability that the withdrawn balls are the same color?

(c) Show that the probability in part (b) is always larger than the one in part (a).

Problem 5. A small village consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children, and 1 has five children.

(a) If one of these families is chosen at random, what is the probability that it has i children, for $i = 1, 2, 3, 4, 5$?

(b) If one of the children is randomly chosen, what is the probability that this child comes from a family having i children, for $i = 1, 2, 3, 4, 5$?

Problem 6. An urn contains 5 red and 6 black balls. Players A and B withdraw balls from the urn consecutively (A draws first, then B, then A, then B, etc.) without replacement until a red ball is selected. The player who selects the red ball wins the game. Find the probability that player A wins.

Problem 7. A forest contains 20 deers, of which 5 are captured, tagged, and released. A month later, 4 of the 20 deers are captured. What is the probability that exactly 2 of these 4 have been tagged?

Problem 8. If it is assumed that all $\binom{52}{5}$ poker hands are equally likely, what is the probability of being dealt

(a) a flush? (A hand is said to be a flush if all 5 cards are of the same suit.)

(b) two pairs? (This occurs when the cards have values a, a, b, b, c, where a, b, c are all distinct.)

Problem 9. There are 30 physicists and 25 chemists attending a certain conference. Three of these 55 people are randomly chosen to take part in a panel discussion. What is the probability that at least one chemist is chosen?

Problem 10.

(a) If n people, including A and B, are randomly arranged in a line, what is the probability that A and B are next to each other?

(b) What would the probability be if the people were randomly arranged in a circle?