# MATH/STATS 425, Homework 5 (due Monday, March 3, 2008)

**Problem 1.** Let $X$ be a random variable taking values in the set of natural numbers, $\mathbb{N} = \{1, 2, 3, \ldots\}$, whose probability mass functions are of the form $p(k) = C2^{-k},\ k = 1, 2, \ldots$.

(a) Find the value of the constant $C$.

(b) $P(X > 1)$;

(c) $P(X \text{ is even})$;

**Problem 2.** For a nonnegative integer-valued random variable $X$, show that

$$E[X] = P(X > 0) + P(X > 1) + P(X > 2) + P(X > 3) + \cdots .$$

**Problem 3.** Let $X$ be a random variable having expected value $\mu$ and variance $\sigma^2$. Find the expected value and the variance of the random variable

$$Y = \frac{X - \mu}{\sigma} .$$

**Problem 4.** Airlines find that each passenger who reserves a seat fails to turn up with probability $\frac{1}{10}$ independently of the other customers. So Teeny Weeny Airlines (TWA) always sell 10 tickets for their 9-seat airplane, while Blockbuster Airways (BA) always sell 20 tickets for their 18-seat airplane. Which is more often over-booked?

**Problem 5.** A man claims to have extrasensory perception. As a test, a fair coin is flipped 8 times, and the man is asked to predict the outcome in advance. He gets 6 out of 8 correct. What is the probability that he would have done at least this well if he had no extrasensory perception?

**Problem 6.** To determine whether or not they have a certain disease, a large number of people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people; whereas, if the test is positive, each of the 10 people will also be individually tested and, in all, 11 tests will be made on this group. Assume that

the probability that a person has the disease is $p$ for all people, independently of each other, and that the pooled test will be positive if at least one person in the pool has the disease.

Let $T$ be a random variable equal to the number of tests needed for a group of 10 people. Find the expected number $E[T]$ of tests necessary for each group. Calculate the numerical values of $E[T]$ for the cases $p = 0.001$, $p = 0.01$, $p = 0.05$, $p = 0.1$, and $p = 0.5$. Discuss briefly the practical applicability of this method (from purely probabilistic point of view).

**Problem 6.** We toss $n$ coins, and each one shows heads with probability $p$, independently of the others. Each coin which shows heads is tossed again. Let $Y$ be the number of heads resulting from the second round of tosses. Find the probability mass function, $f_Y(k)$, the expectation, $E[Y]$, and the variance, $\mathrm{var}(Y)$, of the random variable $Y$.