MATH/STATS 425, Homework 6 (due Monday, 3/10/2008)

Problem 1. Suppose that the average number of cars abandoned weekly on a certain highway is 2.2. Approximate the probability that there will be:

(a) no abandoned cars in the next week;

(b) at least 2 abandoned cars in the next week.

Problem 2. Suppose that the number of misprints occurring in on a page is a Poisson random variable with parameter $\lambda = 3$.

(a) Find the probability that 3 or more errors occur on page 12.

(b) Find the probability 3 or more errors occur on page 12 if it is known that at least one error occurred on page 12.

Problem 3. The suicide rate in a certain state is 1 suicide per $100,000$ inhabitants per month.

(a) Find the probability that in a city of $400,000$ inhabitants within this state, there will be 1 or more suicides in a given month.

(b) Find the probability that there will be less then 1 suicides in this city in 2 months.

(c) What is the probability that there will be at least 2 months during the year that will have 1 or more suicides?

(d) Counting the present month as month number 1, what is the probability that the first month to have 1 or more suicides will be month number i, $i \geq 1$?

Problem 4. This problem is an illustration that Poisson random variable is sometimes a good approximate of the number of successes even when the trials are only 'weakly independent'. Suppose that there are N men in a room. Each of them throws his into the center of the room. The hats are mixed up, then each man randomly selects a hat. In Chapter 2, we considered this matching problem and showed that

$$P(\text{none of the men selects his own hat}) = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + \frac{(-1)^N}{N!}.$$

In this problem, we will compute it approximately using Poisson random variable.

(a) Let E_i denote the event that person i selects his own hat, for $i = 1, 2, \cdots, N$. Compute $P(E_i)$ and $P(E_i|E_j)$, $i \neq j$.

(b) Show that E_i and E_j are not independent for $i \neq j$.

(c) Even though E_j's are not independent, use the Poisson random variable with parameter $\lambda = 1$ as a approximate of the number of 'successes' (=the number of matchings) to compute

$$P(\text{none of the men selects his own hat})$$

approximately. Discuss that this is indeed close to $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + \frac{(-1)^N}{N!}$. when N is large.