

## MATH/STATS 425, Homework 7 (Do NOT hand in)

**Problem 1.** The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter  $\lambda = 5$ . Suppose that a new wonder drug has just been marketed that reduces the Poisson parameter to  $\lambda = 3$  for 75 percent of the population. For the other 25 percent of the population the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial for the person?

**Problem 2.** Two athletic teams play a series of games; the first team to win 4 games is declared the overall winner. Suppose that team A is stronger than team B and wins each game with probability .6, independent of the outcomes of the other games.

- (a) Find the probability that team A wins the series in exactly  $i$  games for  $i = 4, 5, 6, 7$ .
- (b) Let  $P_1$  be the probability that team A wins in a series of 7 games; the first team to win 4 games is declared the overall winner. Let  $P_2$  be the probability that team A wins in a series of 3 games; the first team to win 2 games is declared the overall winner. Is  $P_1$  larger than  $P_2$ ?

**Problem 3.** If  $X$  is a geometric random variable, show analytically that

$$P\{X = n + k | X > n\} = P\{X = k\}.$$

Then, give a verbal argument IN COMPLETE SENTENCES, using the interpretation of a geometric random variable as to why the preceding equation is true.

**Problem 4.** An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 of them are white and two are black then we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white. What is the probability that we shall make exactly  $n$  selections? How many selections on average do we make?

**Problem 5.** The probability density function of  $X$ , the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2}, & x > 10, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $P((X > 15))$ .
- (b) What is the probability that of 3 such types of devices at least one will function for at least 15 hours?