MATH/STATS 425, Homework 8 (Due Monday 3/31)

Problem 1. Trains headed for destination A arrive at the train station at 15-minute intervals starting at 7 A.M., whereas trains headed for destination B arrive at 15-minute intervals starting at 7:05 A.M.

(a) If a certain passenger arrives at the station at a time uniformly distributed between 7 and 8 A.M. and then gets on the first train that arrives, what proportion of time does he or she go to destination A?

(b) What if the passenger arrives at a time uniformly distributed between 7:10 and 8:10 A.M.?

Problem 2. A point is chosen at random on a line segment of length L. Interpret this statement and find the probability that the ratio of the shorter to the longer segment is less then $\frac{1}{4}$.

Problem 3. Two types of coins are produced at a factory: a fair coin and a biased one that comes up heads 55 percent of the time. We have one of these coins but do not know whether it is a fair coin or a biased one. In order to ascertain which type of coin we have, we shall perform the following statistical test: We shall toss the coin 1000 times. If the coin lands on heads 525 or more times, then we shall conclude that it is a biased coin, whereas, if it lands heads less than 525 times then we shall conclude that it is the fair coin. If the coin is actually fair what is the probability that we shall reach a false conclusion? What would it be if the coin were biased? State clearly any assumptions you use?

Problem 4. One thousand independent rolls of a fair die will be made. Compute an approximation to the probability that number 6 will appear between 150 and 200 times inclusively. If number 6 appears exactly 200 times, find the probability that number 5 will appear less then 150 times. State clearly any assumptions.

Problem 5. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \quad x \geq 0.$$

Compute the expected lifetime of such a tube.

Problem 6. If X is uniformly distributed over $(-1, 1)$, find $E[|x|]$.

Problem 7. The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years? Explain your reasoning.