

MATH 526 Homework 1. Due Tuesday 1/20

Problem 1. Find the function $f(n)$, $n = 0, 1, 2, \dots$, that satisfies

$$\begin{aligned}f(n) &= \frac{1}{3}f(n-1) + \frac{1}{3}f(n+1) + \frac{1}{3}f(n+2), & n \geq 1, \\f(0) &= 0, \\ \lim_{n \rightarrow \infty} f(n) &= 1.\end{aligned}$$

Problem 2. Find the functions $x(t)$, $y(t)$ that satisfy

$$\begin{aligned}x'(t) &= y(t) - x(t), \\y'(t) &= 3x(t) - 3y(t),\end{aligned}$$

and $x(0) = 0$, $y(0) = 1$.

Problem 3. Let $0 < p < 1$. Find the general solutions to the difference equation

$$f(n) = (1-p)f(n-1) + pf(n+1).$$

Do not hand in the solutions to the following problems.

Problem 4. The Fibonacci numbers F_n are defined by

$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 2,$$

and $F_1 = F_2 = 1$. First a few numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots . Find a formula for F_n .

Problem 5. Prove by induction that

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

and

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 6. Recall the geometric series

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}, \quad |r| < 1.$$

Also recall the exponential series

$$1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots = e^x, \quad -\infty < x < \infty.$$