Consider

\[ \frac{d^2y}{dx^2} + (6\lambda + 9)y = 0 \]

\[ (\lambda + 3)^2 \quad \text{if} \quad \lambda = -3, \quad -3, -3 \]

Can we write a solution

\[ y(x) = c_1 e^{-3x} + c_2 e^{-3x} = (c_1 + c_2) e^{-3x} \]

We need another linearly independent solution.

\[ y(x) \]

\[ \begin{array}{c}
\text{No} \\
\text{EC}
\end{array} \]
OPERATOR IDENTITY II

\[(D-a)^n x^k e^{ax} = 0 \quad k = 0, 1, \ldots, n-1\]

i.e. operator \((D-a)^n\) annihilate functions \(e^{ax}, xe^{ax}, x^2 e^{ax}, \ldots, x^{n-1} e^{ax}\)

\[\text{Proof} \]

\[ (D-a) x^k e^{ax} = x^{k-1} e^{ax} + x^k a e^{ax} - a x^k e^{ax} = x^{k-1} e^{ax} \]

\[ D( x^k e^{ax} ) \]

We see that when operator \(D-a\) acts on \(x^k e^{ax}\), the result is the same as it would be if we just differentiated the term \(x^k\) and multiplied then by \(e^{ax}\).
\[(D-a)^2 \times e^{ax} = (D-a)(kx^{k-1}e^{ax}) = k(k-1)x^{k-2}e^{ax}\]

\[(D-a)^3 \times e^{ax} = k(k-1)(k-2)x^{k-3}e^{ax}\]

\[
(D-a)^k \times e^{ax} = k(k-1)(k-2)\ldots \left(\frac{k-(k-1)}{k!}\right)e^{ax} = 1
\]

\[(D-a)^{k+1} \times e^{ax} = 0\]

We have shown that \((D-a)^n\) annihilates \(x^ke^{ax}\) whenever \(n\) is a non-negative integer between 0 and \(n-1\): \(k=0,1,\ldots,n-1\)

\((D-5)^5\) annihilates \(e^{5x}, xe^{5x}, x^2e^{5x}, x^3e^{5x}\)

\((D+1)^6\) annihilates \(e^{-x}, xe^{-x}, x^2e^{-x}, x^3e^{-x}, x^4e^{-x}, x^5e^{-x}\)
\[(D^2 - 4y^2)^3 = (D - 4y)^3 (D + 4y)^3\] annihilates \[e^{rx}, xe^{rx}, x^2e^{rx}, e^{-rx},\]
\[xe^{-rx}, x^2e^{-rx}\]

Returning to \[\frac{d^2y}{dx^2} + G \frac{dy}{dx} + 9y = 0\]
\[(D^2 + 6D + 9)y = 0\]
\[(D + 3)^2 y = 0\]
\[-3, -3\]

Solutions: \[e^{-3x} \text{ and } xe^{-3x}\]

General solution: \[y(x) = C_1 e^{-3x} + C_2 x e^{-3x}\]
\[ (D-\alpha)e^x = 0 \]

\[ (D-\beta)e^x = 0 \]

\[ (D-\gamma)e^x = 0 \]

\[ y(x) = C_1e^{\alpha x} + C_2xe^{\alpha x} + C_3xe^{\beta x} + C_4xe^{\gamma x} + C_5e^{-\alpha x} + C_6xe^{-\alpha x} + C_7e^{-\gamma x} \]