Homework # 1, Due Tuesday, September 11

1. **Problem.** Let $A$ and $B$ be events with probabilities $P(A) = 2/3$ and $P(B) = 3/5$. Find the largest and the smallest values of $P(A \cap B)$.

2. **Problem.** A line of 100 passengers is arranged to board a 100-seat airplane. The first passenger lost his boarding pass, so he takes a random seat. Each next passenger walks up to his assigned seat, sits there if the seat is free, or takes a random available seat if the seat is occupied. What is the probability that the last passenger will sit in his assigned seat?

3. **Problem.** Let $A, B$ and $C$ be events. Using the axioms of probability, show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

4. **Problem.**
   a) I shuffle the standard deck of 52 cards, take the top card, and without looking at it, put it away. What is the probability that the next card is an ace?
   b) I look at the card I put away and see that it is the King of spades. What is the probability that the next card is an ace?

5. **Problem.** There are four coins in a box: one double-headed, one double-tailed and two fair coins.
   a) I close my eyes, pick a coin from the box at random and toss it. What is the probability that the coin lands heads?
   b) I open my eyes and see that the coin indeed landed heads. What is the probability that it is the double-headed coin?
   c) I close my eyes and toss the same coin again. What is the probability that the coin lands heads?
   d) I open my eyes and see that the coin indeed landed heads. What is the probability that it is the double-headed coin?
   e) I throw the coin away, close my eyes, pick another coin from the box at random and toss it. What is the probability that the coin lands heads?