

Math 425, Section 6, Fall 1999: Introduction to Probability

First Mid-Term: October 8, 1999

Answer all questions, and answer in complete sentences. Points for each question are marked. Calculators and notes are not allowed on this exam.

1. Five cards are selected at random from a standard deck of 52 playing cards.

- (i) What is the probability of getting a “full house”, *i.e.*, three cards of one denomination and two of another denomination?
- (ii) What is the probability of getting exactly two pairs?

[20]

2. One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from the lot and then tossed, turns up heads six times in a row, what is the probability that it is the two-headed coin?

[20]

3. (a) When are two events E and F in the sample space S independent? When are three events E, F, G in S independent?

(b) Let us flip a fair coin three times, and let A_{12} be the event that the first and second flips come out the same (*i.e.*, both heads or both tails). Similarly for events A_{13}, A_{23} . Are A_{12} and A_{13} independent? Are A_{12}, A_{13}, A_{23} all independent?

[20]

4. In light of the gambler’s ruin problem, discuss the following two quotations:

- (a.) “Millionaires should always gamble, poor men never.” [John Maynard Keynes]
- (b.) “If I wanted to gamble, I would buy a casino.” [J. Paul Getty]

[Note: John Maynard Keynes was a distinguished economist; J. Paul Getty a famous oil tycoon.]

[15]

(please turn over for 5th question)

5. (a.) Suppose we flip a biased coin with probability of getting a head equal to p many times in a row, independently. What is the probability that you will flip a head for the *first* time in this sequence of flips on the n -th flip?

(b.) Suppose that a cereal maker offers 5 different premiums in single serving packages of its cereal, and you buy one such package every morning. You want to collect all five premiums! You are equally likely each morning to get any one of the five premiums in your cereal. You have just gotten your third distinct premium. What is the probability that you will get your fourth kind within the next three days?

[25]

Quotation:

Lastly, numbers are applicable even to such things as seem to be governed by no rule, I mean such as depend on chance: the quantity of probability and proportion of it in any two proposed cases being subject to calculation as much as anything else. Upon this depend the principles of game. We find sharpers as know enough of this to cheat some men that would take it very ill to be thought bubbles; and one gamester exceeds another, as he has a greater sagacity and readiness in calculating the probability to win or lose in any particular case. To understand the theory of chance thoroughly, requires a great knowledge of numbers, and a pretty competent one of Algebra.

John Arbuthnot

An essay on the usefulness of mathematical learning

25 November 1700