Math 425, Section 1, Winter, 1997: Intro to Probability.

Second Mid-term, March 21, 1997.

Please answer all question, *with complete sentences.* You are allowed to use a calculator, as well as the sheet of distribution functions and Φ-values provided. (Please note that one more distribution is given at the end of the questions below, which did not appear on the sheet given.)

1. Let $\mathbf{X}$ be a random variable whose probability density function f is given by $f(x) = xe^{-x}$, for $x \geq 0$, and 0 otherwise. What are the cumulative distribution function and expectation of $\mathbf{X}$? [15]

2. Suppose we flip a coin 20 times and get 7 heads. Suppose we also don't know the bias of the coin we are flipping. What probability p for a head would maximize the chance that 7 heads should appear out of 20 flips of the coin? (This is the first example of the statistical technique of *maximal likelihood estimation.*) [15]

3. A pond contains 100 fish, of which 30 are carp. If 20 fish are caught, what are the mean and the variance of the number of carp among the 20 fish caught? What assumptions are you making? [20]

4. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time *uniformly distributed* between 10 and 10:30.

(a) What is the probability that you will have to wait longer than 10 minutes?

(b) If at 10:15 the bus has not yet arrived, what is the probability that you will wait at least another 10 minutes? [25]

5. For each of the random variables $\mathbf{X}$ below, determine the type of distribution (*i.e.,* normal, uniform, Poisson, hypergeometric, binomial, negative binomial, *etc.*) which best models $\mathbf{X}$. Where possible, give the values of the parameters of the distribution chosen. Give reasons for your choice of distribution. [25]

(a) The average height of professors at a certain college is 68 inches, and the mean squared deviation (or variance) from this average is 2. $\mathbf{X}$ is the height of a randomly chosen professor.

(b) As part of a grand opening promotion, a department store has advertised that every one thousandth purchase made on opening day will be given to the customer for free. The store expects five purchases to be made every minute. $\mathbf{X}$ is the time from opening until the first purchase is given away.

The Hypergeometric Distribution. $p(i) = \binom{M}{i}\binom{N-M}{n-i}\Big/\binom{N}{n}$, $M \leq N, 0 \leq i \leq n$. The expectation is $n\frac{M}{N}$, and the variance is $\frac{N-n}{N-1} \cdot n\left(\frac{M}{N}\right)\left(1 - \frac{M}{N}\right)$.