Many variational models for image segmentation are non-convex and are known to possess local minima. These models are often solved by iterative energy minimization; successful results therefore require a good initial guess. An example is the two-phase piecewise constant Mumford-Shah model that aims to separate a given image $f(x)$ into two regions: Foreground and background.

Motivated by previous work of T. Chan and L. Vese, we developed an algorithm that helps avoid local minima of this model. When the average intensities $c_1$ and $c_2$ of the foreground and background in image $f(x)$ are known, it is guaranteed to find a global minimizer. The idea is to minimize the following convex energy:

$$u(x) = \arg\min_{0 \leq v(x) \leq 1} \int |\nabla v| + \lambda \int [(c_1 - f)^2 - (c_2 - f)^2] v\,dx$$

Using observations made in previous works of G. Strang, we proved that cross-sections of $u(x)$ depict the boundary between foreground and background regions of a global minimizer of the original model. Our technique thus significantly reduces the dependence of results on initial guess.