

Math 651: Variational Models in Image Processing and Computer Vision

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Course description: This course will introduce students to a number of problems in image processing and computer vision, and describe how they can be tackled by modern techniques based on calculus of variations and partial differential equations. We will focus especially (but not exclusively) on image reconstruction (denoising, deblurring, inpainting, as well as some inverse problems) and image segmentation. These procedures are fundamental in many applications, such as medical imaging and target recognition. Numerical solution of the models, which involve minimizing appropriate energies (often by solving associated partial differential equations) will be a major concern of the course: A variety of numerical techniques for this purpose, including level set and diffuse interface methods for evolving curves and surfaces, will be introduced and covered in detail. In addition, important theoretical questions about the various models and how they have been answered will be presented.

Prerequisites: Some coursework in (or willingness to learn) partial differential equations and numerical methods. Ability to program in (or willingness to learn) a language such as Fortran, C, or Matlab.

Grading: There will be short problem sets and simple programming tasks.

Textbook:

Aubert, Gilles; Kornprobst, Pierre. *Mathematical Problems in Image Processing. Partial Differential Equations and the Calculus of Variations*. Applied Mathematical Sciences #147. Springer Verlag. New York, 2006.

Tentative Outline

1. Mathematical preliminaries (Chapter 2 of textbook).

- Some basic techniques in the calculus of variations.
- Functions of bounded variation.
- Variational approximation: Gamma convergence.
- Geometry via level sets.

2. Image denoising and deblurring (Chapter 3 of textbook).

- Linear diffusion.
- Anisotropic diffusion: Motion of isocontours by mean curvature.
- The total variation model of Rudin, Osher, and Fatemi in detail:
 - Applications: Denoising, deconvolution, inpainting, inverse problems,...
 - Numerical techniques: Gradient descent, fixed point iteration, convex duality,...

- Analysis: Properties of minimizers, some exact minimizers, total variation flow.
- Variants of the ROF model: Improvements for preserving features (texture, corners, contrast), reducing artifacts such as staircasing, . . .
- The Perona-Malik method and its regularized versions.
- Some fourth order models.

3. **Image segmentation** (Chapter 4 of textbook)

- Active contours model of Kass, Witkin, and Terzopoulos: Segmentation via “snakes” (curve evolution).
- Geodesic active contours; implementation using level-sets.
- Global active contours method of Cohen & Kimmel.
- Blake and Zisserman’s discrete segmentation energies.
- The Mumford-Shah model of image segmentation, in detail:
 - Piecewise constant versions: Minimal partition problems (Chan & Vese):
 - * Level set based formulation,
 - * Recent alternative solution techniques.
 - Ambrosio and Tortorelli’s approximation: Diffuse interface formulation.
- Some theory:
 - Existence of minimizers.
 - Properties of minimizers (Regularity, Euler-Lagrange equations, behavior at junctions, etc.)
 - Sufficient conditions for minimizers and some exact solutions: The calibration method.
 - Precise relation with Blake and Zisserman’s energies.
- Other variants of Mumford-Shah: Curvature dependent functionals, segmentation with depth, applications to inpainting.

4. Discussion of a few other vision problems.