1. Let \( X = \left\{ \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & 0 \end{bmatrix} \right\} \cong \mathbb{C}^8 \). Describe a “cluster structure” in the polynomial ring \( \mathbb{C}[X] = \mathbb{C}[z_{11}, z_{12}, \ldots, z_{32}] \) in which

- the set of cluster variables includes \( z_{11}, z_{12}, z_{21}, z_{22}, z_{23}, z_{32}, \Delta_{12,13}, \Delta_{13,12} \);
- the set of coefficient variables includes \( z_{13}, z_{31}, \Delta_{12,12}, \Delta_{12,23}, \Delta_{23,12} \);
- the set of exchange relations includes the relations of the form \( z_{ik}z_{j\ell} = \Delta_{ij,k\ell} + z_{i\ell}z_{jk} \) as well as the relations

\[
\begin{align*}
z_{12}\Delta_{12,13} &= z_{11}\Delta_{12,23} + z_{13}\Delta_{12,12}, \\
z_{21}\Delta_{13,12} &= z_{11}\Delta_{23,12} + z_{31}\Delta_{12,12}, \\
z_{22}\Delta_{13,12} &= z_{21}\Delta_{12,23} + z_{23}\Delta_{12,12}.
\end{align*}
\]

2. A minor \( \Delta_{I,J} \) is called solid if both \( I \) and \( J \) consist of several consecutive indices. A matrix \( z \in \text{SL}_n(\mathbb{R}) \) has \( n^2 \) solid minors \( \Delta_{I,J} \) such that \( I \cup J \ni 1 \). Show that \( z \) is totally positive if and only if all these \( n^2 \) minors are positive. (You may rely on the fact that \( z \) is totally positive if and only if all flag minors of both \( z \) and its transpose are positive.)

3. Exercise 2.2.3, generalized to an arbitrary positive integer value of the parameter \( k \) that denotes the number of vertices of the quiver that are placed along each side or diagonal of the given convex polygon.

4. Prove Proposition 2.3.2.

5. Exercise 2.6.5.


7. Exercise 2.6.7.

8. Exercise 2.6.10.

9. Classify 3-vertex quivers of finite mutation type. (Note that here frozen vertices are allowed, unlike in the classification theorem stated in class.)

10. Exercise 2.7.9.