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ANDREI ZELEVINSKY, 1953–2013

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Andrei Zelevinsky passed away on April 10, 2013, two weeks before a conference that was intended as a celebration of his 60th birthday. He was born in Moscow on January 30, 1953, and lived there until 1990. From 1991 until his death, he was a professor at Northeastern University in Boston. He was a Managing Editor of *Transformation Groups* during 2005–2011.

Andrei was one of the best mathematicians of his generation. His mathematical talents manifested themselves early on, so the choices of where to study were rather clear. In 1969, he graduated from Moscow's celebrated mathematical school No. 2, and entered the math department at Moscow State University. As a member of the USSR International Mathematical Olympiad team, he was directly granted university admission, bypassing entrance examinations. Already in high school, Andrei met his future *de facto* graduate advisor Joseph Bernstein, who was actively involved in high school math competitions. One year after entering the university, Andrei had another encounter of pivotal importance, which he later described in [M3]: he met Israel M. Gelfand, who soon became his mathematical mentor. Gelfand brought Andrei to his seminar, then one of the principal focal points of Moscow's mathematical life.

Very few of Andrei's classmates became research mathematicians. In hindsight, the main reason seems clear: a mathematician's life—in any country—is rather hard, compared to other careers available to intellectually gifted young people. It is fraught with disappointments and self-doubt; failures are frequent, rewards rare. Andrei, however, was happy in his professional life: his reward was mathematics itself, the subject he loved dearly.

The mathematical problems that appealed to Andrei were usually very concrete

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and combinatorial in nature. With the fascination of a dedicated clockmaster, he would meticulously scrutinize the seemingly minute details of some complicated combinatorial structure, eventually uncovering the hidden jewels lying within. The discovery of cluster algebras was made in this way.

While still in high school, Andrei predicted that he would work in combinatorics. This prediction proved both right and wrong. His main subject of research was representation theory but his style was always very combinatorial, long before the term "Combinatorial Representation Theory" was introduced.

In mathematics, Andrei had many teachers and mentors. He considered himself I. M. Gelfand's "mathematical grandson" [M3], since J. Bernstein was a student of I.M. Besides being an active member of Gelfand's seminar, Andrei participated in other Moscow seminars of the 1970s and 1980s, in particular those run by Yuri Manin, Vladimir Arnold, and Alexandre Kirillov (Andrei's official Ph.D. advisor). All of those people taught him a lot — and he returned the favor by teaching others.

Andrei liked to teach, and had a talent for it. His lucid style combined precise general statements with illuminating concrete examples. Through his lectures, his writings, and his mentorship, he influenced a great many mathematicians. One of the highlights of his teaching career was a stint at the Jewish People's University in Moscow (see [M1]), an underground educational outfit run by Moscow mathematicians for the sake of young Jewish students who were denied entry into Moscow State University because of its anti-Semitic policies of the time.

In 1990, Zelevinsky moved with his family to the United States. After a year at Cornell, he accepted a job at Northeastern, and settled in Sharon, MA, a community he called home for the rest of his life. While many people found the experience of immigration to be stressful, Andrei acclimated to his new environment very quickly. He felt at ease with the ways of Western academia, and his research, already superb in Moscow, truly blossomed in the United States. Defying conventional wisdom, he obtained some of his best results after turning 45!

Andrei was an exemplary citizen of the worldwide mathematical community. He worked tirelessly as a journal editor; as a conference and seminar organizer; and as a mentor, both formal and informal, to young mathematicians. His blog [R1], with comments on matters mathematical, non-mathematical, and meta-mathematical, was popular with many readers who appreciated its moderate tone, rational analysis, and kind humor. At Northeastern, Andrei's reputation was stellar. His opinions, always frank but never confrontational, were universally respected. He never sought a personal advantage. He loved teaching, and he taught more than he had to, organizing a seminar for his graduate students apart from his teaching load.

Andrei's accolades included a Humboldt Research Award (2004) and a University Distinguished Professorship at Northeastern, bestowed posthumously in 2013. He served on the Scientific Advisory Board of MSRI in Berkeley, and on the editorial boards of *Advances in Mathematics*, *Algebra & Number Theory*, *IMRN*, *Journal of Algebraic Combinatorics*, *Selecta Mathematica*, and *Transformation Groups*.

In Fall 2012, MSRI hosted a semester-long Cluster Algebras program, attended by dozens of researchers from all over the world. Andrei Zelevinsky was one of the key participants in the program. He was actively involved in several research projects; co-organized the main topical workshop; and served, as always, as a great mentor to young mathematicians. He fell suddenly ill in December, never to recover.

Andrei is survived by his parents Vladlen and Natalia, his wife Galina, his children Katya and Leo, and his grandchildren Gregory and Julia. Additional biographical information can be found in a *Wikipedia* article [R3] and the sources referenced therein. A collection of remembrances (in Russian) appeared in [R6].

While Andrei Zelevinsky's life was cut short too early, we are consoled by knowing that he was a happy person. He will live on in our memory; in the memory of his family, friends, colleagues, and students; and through the beautiful mathematics that he helped create.

Andrei Zelevinsky's mathematical research

The starting point of Andrei's research activity was his work with J. Bernstein on p-adic groups. It began in 1973 at the instigation of I. M. Gelfand, who suggested that they write a survey on *representation theory of reductive p-adic groups*, a new and important subject whose recent developments were introduced to the Moscow mathematical community in a series of lectures by André Weil.

Andrei and Joseph worked on the project for about two years. In some sense they learned the theory from scratch, and redeveloped some of its foundations. To make the paper more accessible, they restricted the treatment to the general linear groups $\mathrm{GL}(n;F)$. The survey [S1] was based on results by Harish-Chandra, Jacquet, and Gelfand–Kazhdan; it also contained many original results. It was essentially a textbook describing the fundamental structures of the theory.

While working on [S1], the authors realized that they could prove several new results; see [4]. The most important advances were the notion of the normalized Jacquet functor, the Geometric Lemma, and the proof of the irreducibility criterion. Later on, combining this technique with elaborate combinatorial arguments, Andrei gave a complete classification of irreducible representations of the groups $\mathrm{GL}(n;F)$ in terms of cuspidal representations (the Zelevinsky classification [5]).

In the case of $\mathrm{GL}(n)$, the Geometric Lemma implied that the K-groups of the category of representations form a Hopf algebra. This holds both for p-adic fields and for finite fields. In the latter case, the Hopf algebra operations preserve the natural positivity structure. Andrei realized that these properties can be elegantly axiomatized via the notion of a *PSH algebra*. He then showed [B1] that all PSH algebras can be reduced to one particular example, the representation ring of symmetric groups, and, moreover, that all basic results about this ring (i.e., the theory of symmetric functions) can be deduced within this framework. Later, in [9], Andrei and Tonny Springer combined this approach with Brauer theory to obtain a complete classification of irreducible representations of groups $\mathrm{GL}(n)$ over finite fields.

Andrei retained a deep interest in the theory of symmetric functions, including both its combinatorial and representation-theoretic aspects, throughout his mathematical career. His original ideas and ingenious constructions (see in particular [R4], [B1], [6], [S4], [S7]) had a profound influence on the subject.

Having had Bernstein and Kirillov as advisors, Andrei was, from early on, exposed to the influence of I. M. Gelfand. One fundamental theme in Gelfand's mathematical pursuits was the exploration of the relationships between representation theory and various areas of classical mathematics, such as combinatorics and the theory of special functions. Starting in the mid-1980s, Andrei was deeply involved in a series of collaborative projects with Gelfand and other members of his circle, developing what has quickly become a vast research program centered around *higher-dimensional analogues of hypergeometric functions* [15], [16], [18], [22], [23], [32], [33]. The main point of this program is the interplay between various approaches to special functions: in terms of power series, differential equations, or integral representations. In particular, the study of Grassmannian hypergeometric functions [15], [16], defined as integrals of products of powers of linear functions, led to new perspectives on hyperplane arrangements, convex polytopes, and matroids, and stimulated much of the subsequent research in the area. The important class of functions nowadays called A-hypergeometric functions (or GKZ hypergeometric functions) was introduced in his paper [18] with Gelfand and M. I. Graev. Hypergeometric systems of differential equations, studied in [18], [22], provided important explicit examples of holonomic systems of linear PDE (and, using a more algebraic language, of holonomic D-modules). These systems are now a standard tool in the study of periods of algebraic hypersurfaces, and in particular of many types of Calabi-Yau varieties appearing in mirror symmetry.

Relations between hypergeometric functions and toric varieties inspired Andrei's work with Gelfand and Mikhail Kapranov on *discriminants of polynomials in many variables*. This work, summarized in their influential book [B2], exhibited new properties of very classical algebraic concepts and had substantial impact on algebraic geometry and the theory of convex polytopes. A central new concept that emerged from this work is that of the *secondary polytope* $\Sigma(Q)$ associated to a given (primary) convex polytope Q. The vertices of $\Sigma(Q)$ correspond to (well-behaved) triangulations of Q, its edges correspond to elementary modifications (Pachner moves), and so on. Gelfand, Kapranov, and Zelevinsky found that the secondary polytope $\Sigma(Q)$ is intimately related to the Newton polytope of the discriminant of a generic polynomial whose Newton polytope is Q.

One recurring theme in Zelevinsky's work in representation theory was the study of linear bases in representation spaces. It began with his 1984 papers [10], [12], joint with I. M. Gelfand. The pioneering idea of [13] was to take the coordinate algebra $\mathcal{A}_G$ of the base affine space of a reductive/semisimple Lie group G (which carries all irreducible representations of G), and try to construct a "good" basis in $\mathcal{A}_G$ compatible with the lattice of PRV-subspaces in $\mathcal{A}_G$. The main goal was to get a rule for *tensor product multiplicities* for G—which would follow from the construction of such a basis because the dimension of a PRV-subspace $V_\lambda(\mu-\nu,\nu)$ equals the multiplicity of the simple module V_μ in the tensor product $V_\lambda \otimes V_\nu$. In joint papers with Gelfand [14] and Vladimir Retakh [20], this program was succesfully carried out for $G=\mathrm{SL}_3$ and $G=\mathrm{Sp}_4$, respectively. Around 1988, Andrei computed (but did not publish) the good basis for $G=\mathrm{SL}_4$. Its slight generalization appeared five years later in [40], within the framework of *string bases*.

Several major developments followed in 1989–90: the proof by O. Mathieu of the existence of good bases; the discovery of canonical bases by G. Lusztig; and the introduction of crystal bases by M. Kashiwara (their duals are good bases). All of this gave a boost to the tensor product multiplicity problem, which was extensively studied by Andrei and his student Arkady Berenstein in a series of papers [21], [27], [35]. Together, they developed the machinery of convex polyhedral cones relevant to this problem, and ultimately arrived at a complete solution in [60].

Andrei's work on good bases elucidated the deep relationships between the geometry of the group G and its (or more precisely, its Langlands dual's) representations. His exploration of this relationship in a series of joint papers with A. Berenstein and S. Fomin written in the mid-to-late 1990s culminated in the solution of a class of factorization problems in G via the technique of *Chamber Ansatz* [48], [49], [53], [60], and eventually led to the discovery of cluster algebras.

During the last 13 years of his life, much of Andrei Zelevinsky's research was centered around *cluster algebras*, a surprisingly rich concept conceived by him and Sergey Fomin at the Erwin Scrödinger Institute in Vienna in May 2000; cf. [62]. The original impetus came from the desire to understand, on a concrete combinatorial level, two fundamental Lie-theoretic notions introduced by G. Lusztig: *total positivity* in Lie groups and the (dual) *canonical bases*. Andrei and Sergey discovered that many of the constructions arising in this part of Lie theory can be naturally interpreted in the language of *mutations* applied simultaneously to quivers (or more generally exchange matrices) and to associated *clusters* of ring elements. They recognized the key role played by the *Laurent phenomenon* for cluster mutations [62], [63], and systematically built the foundations of the theory [64], [68], [69], [71], [74]. Papers [69], [71] were co-authored with A. Berenstein.

While the structural theory of cluster algebras is rather beautiful (for example, cluster algebras with finitely many clusters are classified by Cartan matrices of finite type [68]), the concept owes most of its importance to external factors. During the last decade, it found many applications in a variety of contexts throughout mathematics and theoretical physics. In addition to representation-theoretic topics mentioned above, the list of those contexts includes discrete integrable systems and statistical physics; Teichmüller theory and its generalizations; Poisson and symplectic geometry; string theory; tropical geometry; combinatorial invariant theory; and algebraic and polyhedral combinatorics. Remarkably, cluster algebras provide a unifying framework for a wide range of phenomena in these and other settings. See [R5] and the links posted therein.

Another long-term interest of Andrei's was *quiver representations*. With Peter Magyar and Jerzy Weyman, he worked on the problem of classifying the n-tuples of parabolic subgroups $P_1, \ldots, P_n$ in a simple algebraic group G for which the diagonal action of G on $G/P_1 \times \cdots \times G/P_n$ has finitely many orbits. They solved this problem for the special linear [54] and symplectic groups [55] using representations of quivers (resp. ordinary or symmetric). For other G, the problem remains open.

Papers [77], [79] with Harm Derksen and Weyman dealt with representations and mutations of *quivers with potentials*. A potential S on a quiver Q (i.e., a linear combination of oriented cycles) gives rise to a Jacobian algebra defined by relations given by partial derivatives of S with respect to the arrows of Q. For an arbitrary vertex of Q, the authors introduced a natural notion of mutation of the potential and of representations of the Jacobian algebra, thus obtaining a far-reaching generalization of reflection functors of Bernstein, Gelfand, and Ponomarev. This general theory was applied in [79] to solve several important conjectures on cluster algebras posed in [74], taking advantage of geometric interpretations of basic cluster-algebraic notions in terms of Euler characteristics of quiver Grassmannians.

Graduate students of Andrei Zelevinsky (all at Northeastern University): Arkady Berenstein (Ph.D. 1996), Oleg Gleizer (Ph.D. 2001), J. Scott (Ph.D. 2003), Ahmet Seven (Ph.D. 2004), Paul Sherman (M.S. 2004), Giovanni Cerulli Irelli (Ph.D. 2008, co-advised with A. Tonolo), Daniel Labardini-Fragoso (Ph.D. 2010), Thao Tran (Ph.D. 2010), Shih-Wei Yang (Ph.D. 2010), Sachin Gautam (Ph.D. 2011, co-advised with V. Toledano Laredo), Salvatore Stella (Ph.D. 2013).

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This publication list is based on the one posted on A. Zelevinsky's website [R2].

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