

1. (4 points) Alex Artzy is contemplating the purchase of the new *Nano* music player. At one store, the price on Monday is \$199, but by the following Monday it has dropped to \$179. If the price  $P$  is a linear function of time  $t$ , find an equation for the price of the player as a function of time. When is the price below \$100?

*Solution:* Assuming that  $t = 0$  is the first Monday and that  $t$  is measured in days, we know that the linear function goes through the points  $(0, 199)$  and  $(7, 179)$ . So the slope is  $\frac{179-199}{7-0} = -\frac{20}{7}$ . Then, using the point-slope form of a line we have

$$P - 199 = -\frac{20}{7}(t - 0), \quad \text{or}$$
$$P = -\frac{20}{7}t + 199.$$

Therefore the price is equal to \$100 when  $100 - 199 = -\frac{20}{7}t$ , or  $t = \frac{7 \cdot 99}{20} = 34.65$  days. So after 35 days the price is below \$100.

(Clearly, we could also do this with  $t$  being the time in weeks, with  $t = 0$  giving the first Monday and  $t = 1$  the second. Then  $P = 199 - 20t$ , which is perhaps more pleasing from an aesthetic standpoint.)

2. (4 points) Alex suddenly worries that the price might actually be exponential. Assuming the same price data as in (1), find an equation for the price of the player in this case.

*Solution:* In this case, using the same  $t$  as before, the points  $(0, 199)$  and  $(7, 179)$  lie on an exponential, which has the form  $P = P_0 a^t$ . We know that  $P_0 = 199$ , so we can plug in the other point to find  $a$ . This gives  $179 = 199a^7$ , so  $a^7 = \frac{179}{199}$ , and  $a = (\frac{179}{199})^{1/7} \approx 0.985$ . Thus the equation is  $P = 199(0.985)^t$ . (If  $t$  is in weeks a different exponential is of course correct.)

3. (2 points) If  $P(t)$  is the function giving the price of the coveted *Nano* music player as a function of the time in days since the appointed Monday on which Alex first saw the player advertised, what is the meaning of  $P^{-1}(49)$ ?

*Solution:*  $P(t)$  is the price of the player after  $t$  days, so  $P^{-1}$  gives the day on which the price is a given value. Thus  $P^{-1}(49)$  is the day (counting from that memorable Monday) on which the price is \$49.