1. Suppose that the value of a highly-prized silver-plated author-signed calculus textbook is given, in dollars, by 
\[ V(t) = 100(1.05^t - 0.02t), \]
where \( t \) is the number of years from the publication date of the text. At what rate is the value changing seven years after the book’s publication? (3 points)

**Solution:** The rate of change is given by the derivative, which, using shortcuts, is 
\[ V'(t) = 100(\ln(1.05) \cdot 1.05^t - 0.02). \]
Thus after five years, we have 
\[ V'(7) = 100(\ln(1.05) \cdot 1.05^7 - 0.02) \approx 4.87 \text{ dollars/year}. \]
Thus after seven years, the book is appreciating at a rate of $4.87/year.

2. For what values of \( x \) is \( f(x) = 4x^2 - 3x \) both increasing and concave up? (Use your knowledge of derivatives to answer this question—though your calculator may be useful as you work out your answer.) (4 points)

**Solution:** We know that the function is increasing when its derivative is greater than zero. Here, 
\[ f'(x) = 8x - \ln(3) \cdot 3^x, \] so we want \( 8x - \ln(3) \cdot 3^x > 0. \) Similarly, the function is concave up when its second derivative is greater than zero. Taking the derivative of \( f'(x) \), we have 
\[ f''(x) = 8 - (\ln(3))^2 \cdot 3^x, \] so we want \( 8 - (\ln(3))^2 \cdot 3^x > 0, \) or \( 3^x < 8/(\ln(3))^2. \) Taking the natural log of both sides and solving for \( x \), this becomes \( x < \ln(8/(\ln(3))^2), \) or, finding a decimal approximation, \( x < 1.797. \)

Using the derivative condition is, unfortunately, more difficult: it’s not possible to explicitly solve \( 8x - \ln(3) \cdot 3^x = 0 \) for \( x \). So let’s approximate it on a calculator: graphing \( y = 8x - \ln(3) \cdot 3^x, \) we see that \( y > 0 \) for, approximately, \( 0.165 < x < 2.717. \) From the condition on \( f''(x) \), we know that \( x < 1.797, \) so the range of values on which \( f(x) \) is both increasing and concave up is, approximately, \( 0.165 < x < 1.797. \)

3. Given the following data for \( f, g, f' \) and \( g' \),

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<table>
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<tbody>
<tr>
<td>( x )</td>
<td>( f )</td>
<td>( f' )</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
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a. if \( h(x) = f(x) \cdot g(x) \), find \( h'(1). \)

b. if \( p(x) = f(x)/g(x) \), find \( p'(2). \)

(3 points)

**Solution:** We know from the product and quotient rules that
\[ h'(1) = f'(1) \cdot g(1) + f(1) \cdot g'(1) = (-2)(-4) + (6)(3) = 8 + 18 = 26 \]
and
\[ p'(2) = \frac{f'(2) \cdot g(2) - f(2) \cdot g'(2)}{g(2)^2} = \frac{(-1)(-3) - (4)(5)}{9} = -\frac{17}{9}. \]