MATH 115-018 QUIZ 3 / 20 Jan 2005 Name:______________________________

1. Solve for a: $m \cdot 3^{ab} = k \cdot e^a$. (Here, b, k and m are constants.)

Solution: To solve for a, we take a logarithm of both sides. We'll use the natural logarithm because there's a factor of e^a on the right hand side of the equation. We get

$$\ln(m \cdot 3^{ab}) = \ln(k \cdot e^a), \quad \text{or}$$
$$\ln(m) + ab\,\ln(3) = \ln(k) + a.$$

To solve for a, we subtract $a + \ln(m)$ from both sides, to get $ab\,\ln(3) - a = \ln(k) - \ln(m)$. Factoring out the a on the left hand side, $a\,(b\,\ln(3) - 1) = \ln(k) - \ln(m)$, so

$$a = \frac{\ln(k) - \ln(m)}{b\ln(3) - 1}.$$

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2. Suppose that the temperature of an office is given by $Q(t) = Q_0 + a\sin(\frac{\pi}{6}t)$, where Q is in °F and t in hours after 8AM. What are the meaning of Q_0 and a? Suggest some reasonable values for these parameters.

Solution: The parameter Q_0 is the midline, or average, temperature in the office. The maximum temperature deviation from this is a°F. Reasonable values of Q_0 are probably between 65 and 75, and reasonable values of a are probably 5 or less.

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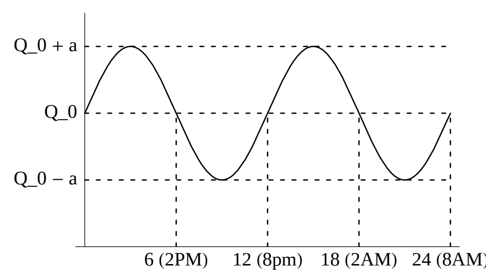
3. Sketch a graph of the temperature function given in (2). Do not substitute values for unspecified parameters.

Solution: We know that the midline of the graph is at Q_0 and that the maximum and minimum values are $Q_0 - a$ and $Q_0 + a$, respectively. The period of the oscillation is the value of t so that $\frac{\pi}{6}t = 2\pi$, or $t = 12$ hours. So the graph must look something like the figure to the right.



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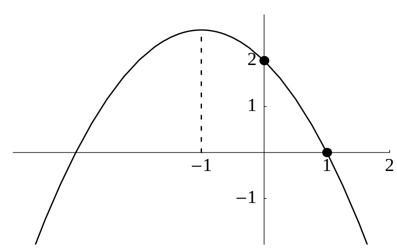
4. Find the equation of a quadratic polynomial $f(x)$ that has each of the following properties:

- $f(1) = 0$,
- $f(0) = 2$,
- $f(x)$ opens down, and
- the vertex of the graph of $f(x)$ is at $x = -1$.



Solution: From the description, the picture that we have in mind is shown in the figure to the right. We know that $f(1) = 0$, so $f(x)$ must have a factor of $x - 1$. It's quadratic, so this means that $f(x) = k(x-1)(x-a)$ for some constants a and k. Then we know that the second zero must be an equal distance on the other side of $x = -1$ from the zero at $x = 1$, so it must be at $x = -3$. Thus $f(x) = k(x-1)(x+3)$. Finally, we plug in $f(0) = 2$ to find $2 = k(-3)$ and $k = -\frac{2}{3}$. The final equation is therefore

$$f(x) = -\frac{2}{3}(x-1)(x+3) = -\frac{2}{3}x^2 - \frac{4}{3}x + 2.$$

Notice that the leading coefficient is negative, so it does open downwards.

Alternate Solution: We know that $f(x) = ax^2 + bx + c$ for some constants a, b and c. Plugging in the second of the two points we're given, we have $f(2) = 0 + c = 2$. Thus, plugging in the first point, we have $f(1) = a + b + 2 = 0$. Finally, we know that the vertex of a parabola is at $x = -\frac{b}{2a}$, so $-1 = -\frac{b}{2a}$, or $b = 2a$. Plugging this back into $a + b = -2$ we get $3a = -2$, so $a = -\frac{2}{3}$ and $b = -\frac{4}{3}$. Then $f(x) = -\frac{2}{3}x^2 - \frac{4}{3}x + 2$.