1. (4 points) The following table gives values for \( f(x) \), \( f'(x) \), \( g(x) \), and \( g'(x) \) at different values of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>2</td>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>2</td>
<td>−1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( g'(x) )</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>−1</td>
</tr>
</tbody>
</table>

a. If \( p(x) = f(g(x)) \), find \( p'(1) \).  

b. If \( q(x) = f(x) \cdot g(x) \), find \( q'(1) \).

**Solution:** For part (a): \( p'(x) = f'(g(x)) \cdot g'(x) \), so \( p'(1) = f'(g(1)) \cdot g'(1) \). \( g(1) = 2 \), so this is \( p'(1) = f'(2) \cdot g'(1) \). Reading these off the table, we have \( p'(1) = 1 \cdot 3 = 3 \).

For part (b): \( q'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \), so \( q'(1) = f'(1) \cdot g(1) + f(1) \cdot g'(1) \). Reading these values off the table, we have \( q'(1) = −1 \cdot 2 + 2 \cdot 3 = 4 \).

2. (2 points) If \( 3xy + \cos(y) + 4 = x^3 \), find \( \frac{dy}{dx} \).

**Solution:** Differentiating both sides of the equation, and remembering to consider \( y \) as a(n implicit) function of \( x \), we get

\[
3y + 3x \frac{dy}{dx} - \sin(y) \frac{dy}{dx} = 3x^2, \quad \text{or}
\]

\[
(3x - \sin(y)) \frac{dy}{dx} = 3x^2 - 3y,
\]

so

\[
\frac{dy}{dx} = \frac{3x^2 - 3y}{3x - \sin(y)}.
\]

3. (2 points) If \( y = 3x - 3 \) is the linear approximation to \( f(x) = x^2 - (a + 1)x + a \) at \( x = 1 \), what is \( a \)?

**Solution:** We know that \( y = 3x - 3 \) must have the same \( y \) value as \( y = f(x) \) at \( x = 1 \), and that its slope must be equal to \( f'(1) \). At \( x = 1 \), the \( y \) value on the line is \( y = 3 - 3 = 0 \), and \( f(1) = 1 - a - 1 + a = 0 \), so that doesn’t tell us anything.

Then \( f'(x) = 2x - (a + 1) \), so \( f'(1) = 2 - a - 1 = 1 - a \). Because \( f'(1) = 3 \), this is \( 1 - a = 3 \), or \( a = -2 \).