1. Suppose that $f(x)$ is an odd function and that $\int_{-2}^{5} 3f(x) + 2\,dx = 23$. (a) What is $\int_{2}^{5} f(x)\,dx$? (b) A passing calculus fan asserts that $f(x) \geq 1$ for $2 \leq x \leq 5$; given the information in this problem, is this assertion correct? (4 points)

**Solution:** (a) We know that $\int_{-2}^{5} 3f(x) + 2\,dx = 3 \int_{-2}^{5} f(x)\,dx + 2\int_{-2}^{5} dx = 3 \int_{-2}^{5} f(x)\,dx + 14$, so $3 \int_{-2}^{5} f(x)\,dx = 9$. Then, because $f(x)$ is odd, we know that $\int_{-2}^{2} f(x)\,dx = 0$, so $3 \int_{2}^{5} f(x)\,dx = 9$, and $\int_{2}^{5} f(x)\,dx = 3$. (b) The assertion is probably not correct. If $f(x) = 1$ for $2 \leq x \leq 5$ then certainly $\int_{2}^{5} f(x)\,dx = 3$. However, if $f(x)$ is not constant for this range of $x$-values, it must take on some values larger (and therefore also smaller) than one for the area to equal exactly three.

2. Suppose that $f''(x)$ is graphed in the figure to the right. Sketch graphs of $f'(x)$ and $f(x)$, indicating on your graphs the locations of the points $x_1$, $x_2$, $x_3$ and $x_4$. (3 points)

**Solution:** The graph of $f''(x)$ (the solid line), $f'(x)$ (dashed), and $f(x)$ (dash-dotted) are shown in the figure. Note that the vertical location of $f'(x)$ is arbitrary; three different possibilities, marked “A”, “B” and “C”, are shown here. The behavior of $f(x)$ does, however, depend on the location of $f'(x)$; the corresponding graphs of $f(x)$ for the three possible $f'(x)$ graphs are shown in the figure. Of course, the vertical location of $f(x)$ is again arbitrary.

3. Find each of the following: (3 points)

(a) $\int 3x^3 - 4 \sqrt{x} \,dx$

(b) $\int \sin(2y) - \frac{1}{\cos^2(y)} \,dy$

(c) $\int \frac{(z-1)^2}{z^2} \,dz$

**Solution:**

(a) $\int 3x^3 - 4 \sqrt{x} \,dx = \frac{3}{4} x^4 - \frac{8}{3} x^{3/2} + C$.

(b) $\int \sin(2y) - \frac{1}{\cos^2(y)} \,dy = -\frac{1}{2} \cos(2y) - \tan(y) + C$.

(c) $\int \frac{(z-1)^2}{z^2} \,dz = \int \frac{z^2 - 2z + 1}{z^2} \,dz = \int \left(1 - \frac{2}{z} + \frac{1}{z^2}\right) \,dz = z - 2 \ln(|z|) - \frac{1}{z} + C$. 