1. Give an explicit formula for a sequence \( s_n \) which has properties that \( s_1 = 2 \), \( \lim_{n \to \infty} s_n = 1 \), and \( s_n < 1 \) for some values of \( n \). What are the first four terms in your sequence? (4 points)

Solution: Answers will vary. One such sequence is \( s_n = 1 - \frac{(-1)^n}{n} \), for which the first four terms are \( s_1 = 2 \), \( s_2 = \frac{1}{2} \), \( s_3 = \frac{4}{3} \) and \( s_4 = \frac{3}{4} \).

2. A daring calculus-loving student leaps from a tree-house located 15 feet above the surface of a trampoline. She then bounces on the trampoline 10 times, attaining a height after each bounce that is \( \frac{3}{2} \) her previous height. (a) write a series giving the total vertical distance she travels after the first bounce from the trampoline (assume that she comes to a stop upon landing on the trampoline for the 10th time), and (b) determine its sum. (4 points)

Solution: After the first bounce she will attain a height of \( (15)(\frac{3}{2}) = 22.5 \) ft, and then will fall an equal distance back to the trampoline, traveling a total distance of 15 ft by the time she lands on the trampoline the second time. After the second bounce she will travel an additional \( 2(15)(\frac{3}{2})^2 = 7.5 (\approx 15\frac{1}{2}) \) ft before landing the third time, and so on (before landing the fourth time, she will travel an additional \( 15(\frac{3}{2})^2 \) ft, etc.). Thus the total distance she travels is \( D = \sum_{n=0}^{8} 15(\frac{3}{2})^n \). This is a finite geometric series with nine terms, so the sum is \( D = 15\frac{1-\left(\frac{3}{2}\right)^9}{1-\frac{3}{2}} \approx 29.94 \) feet.

3. Give an integral that could be used to test the convergence of \( \sum_{n=0}^{\infty} (n-2)e^{-(n-2)} \). Without evaluating the integral, how would it tell you if the series converges or not? (3 points)

Solution: The series is \(-2e^2 - e + 0 + e^{-1} + 2e^{-2} + 3e^{-3} + \cdots\), which has positive decreasing terms for \( n > 3 \). We can thus determine the convergence of the series by considering \( \int_{3}^{\infty} (n-2)e^{-(n-2)} \, dn \). If we were to evaluate the integral (which we could do by substituting \( w = n - 2 \), to get \( \int_{1}^{\infty} we^{-w} \, dw \), which we would then integrate by parts; the value of the integral is \( 2e^{-1} \)), we would know that the series converged if the integral converges (which it does). Similarly, if the integral diverged, so would the series.