1. A calculus student is racing to get to the gateway lab to start taking the Entrance Gateway at the instant the doors open. The student’s velocity, \( v(t) \) (in m/s) is shown in the graph to the right for \( 0 \leq t \leq 8 \) seconds. Write an integral that gives the distance the student travels in those 8 sec, and estimate this distance. (3 points)

Solution: The distance travelled is just \( \int_0^8 v(t) \, dt \). We can approximate this with a left- or right-hand sum, or the average of the two. A left-hand sum is \( \int_0^8 v(t) \, dt \approx 2(0 + 10 + 12.5 + 15) = 75 \) m, and a right-hand sum \( \int_0^8 v(t) \, dt \approx 2(10+12.5+15+15) = 105 \) m. The average is likely to be a more accurate estimate, giving \( \int_0^8 v(t) \, dt \approx 90 \) m.

2. Consider the integral \( \int_0^{3\pi/2} 1 + \sin(x) \, dx \). Let LHS\( (n) \) and RHS\( (n) \) be, respectively, the left- and right-hand sums with \( n \) subdivisions approximating this integral. By looking at a graph (not by evaluating them), place in increasing order the following quantities: LHS\( (3) \), RHS\( (1) \), and \( \int_0^{3\pi/2} 1 + \sin(x) \, dx \). (3 points)

Solution: See the figure to the right. Clearly LHS\( (3) \) > the area under the curve, and at \( x = \frac{3\pi}{2} \), \( 1 + \sin(3\pi/2) = 0 \), so RHS\( (1) = 0 \). Thus we have RHS\( (1) < \int_0^{3\pi/2} 1 + \sin(x) \, dx < \text{LHS}(3) \).

3. Find each of the following derivatives (you need not simplify your answers). (4 points)
   a. \( \frac{d}{dx}(3x\sin(x^2 + 1)) \)
   b. \( \frac{d}{dt}(e^{2t}) \)

Solution: a. \( \frac{d}{dx}(3x\sin(x^2 + 1)) = 3\sin(x^2 + 1) + 6x\cos(x^2 + 1) \).
   b. \( \frac{d}{dt}(e^{2t}) = \frac{2e^{2t}\ln(t) - e^{2t}}{(\ln(t))^2} \) (or, \( = 2e^{2t}(\ln(t))^{-1} - e^{2t}(\frac{1}{2})(\ln(t))^{-2} \)).