1. The following two statements are false. Explain, without any calculation, why they are false. (4 points)
   a. \( \int_{-1}^{1} e^{-x^2} \, dx = -0.746824 \)
   b. \( \int_{-1}^{1} x e^{-x^2} \, dx = 0.632121 \)

Solution: a. We know that \( e^{-x^2} \) is a positive function, so the area between the graph of the function and the x-axis must lie above the x-axis, and the integral must therefore be positive.

b. We know that \( x e^{-x^2} \) is an odd function, so the integral from \(-1\) to \(1\) must be zero.

2. Suppose that the function \( f \) is shown in the figure to the right. If \( F' = f \) and \( F(0) = -1 \), carefully sketch a graph of \( F(x) \) for \( 0 \leq x \leq 4 \). (3 points)

Solution: We know that \( F(0) = -1 \), and that from \( x = 0 \) to \( x = 1 \) its slope is a constant (2). From \( x = 1 \) to \( x = 2 \), \( F(x) \) must continue to increase from \((1, 1)\) to \((2, 2)\) (because the area under \( f(x) \) is one), and its slope must decrease to zero. From \( x = 2 \) to \( x = 3 \) it decreases by one-half to \((3, 2)\) and the slope decreases from zero to \(-1\). From \( x = 3 \) to \( x = 4 \) the slope is a constant \(-1\). This is shown in the figure to the right, below.

3. If the average value of \( f(x) = 9x^2 \) on the interval \( 0 \leq x \leq b \) is 48, what is \( b \)? (3 points)

Solution: The average value we want is \( \frac{1}{b-0} \int_{0}^{b} 9x^2 \, dx = \frac{1}{b} \left( \frac{3x^3}{3} \right)_{0}^{b} = 3b^2 \). Thus we need \( 3b^2 = 48 \), so \( b^2 = 16 \) and \( b = \pm 4 \).