Effectively Using Applied Writing Projects in Undergraduate Mathematics Courses

P. Gavin LaRose

Department of Mathematics
University of Michigan
glarose@umich.edu

3 Aug 2010
Overview

- **Definitions**: what are “Applied Writing Projects”?
- Some **Logic**: why use them?
  
  _a logical example_

- Some **Logistical** and **Practical** matters
- The **Elephant** in the room...  
  
  _...grading_

- Philosophy and Further Directions
  
  _groups, advice, project selection_

- Conclusions and Resources
An Applied Writing Project is... 

- A Really Big *Story Problem*
  - An Applied, (mostly) Real-World,
  - *Multi-step*, often *Multi-part* problem.

- Presented as a *Contract Letter* (or, letter requesting help):
  - *Dear IMC*: The Spate Gallery has recently been approached by an antiques dealer... 
  - *Dear Calculus Students*, I have a problem... 

- Requiring a *Written, Multi-page Solution* (3–7 pages)
- ...and a commensurate amount of work.

...an idiocyncratic definition

(...Gary Larson)

in many undergraduate courses... most?!
Why we might use them

you have too much time on evenings & weekends?

- Applications are **fun** and **motivational**, & show mathematics beyond the boundary of the textbook, as having **importance** and **relevance** in its own right.

- They provide a vehicle to **focus on the substantial material** being covered. . .

- They require **conceptual understanding** and **problem solving**.

- And **writing** can be both **pedagogically sound** and **practically useful**.

  . . . and they can be graded without instructor fatalities
Dear IMC:

The Spate Gallery has recently been approached by an antiques dealer with headquarters in the outskirts of Athens who has offered us first purchase rights to a so-called ancient tapestry. However, we are concerned with its authenticity and so are contacting you to ascertain its age.

The dealer asserts that the tapestry is 2000 years old, having been loomed sometime in the first century B.C. In order to determine the veracity of this claim, our technical analysis department was able to obtain a small portion of the tapestry and has done an analysis of the Carbon-14 present therein. They report the presence of 190 nanograms (ng) of Carbon-14 in the 1 gram sample they took.

As you will, of course, know from your own references on the subject, the amount of C14 present in the environment has varied with time. This amount is constant in living creatures, and then decays (with a half-life of 5,780 years) after they die. The figure below shows the amount (in ng) of C14 (per gram) present in the environment for the past 2,100 years. This was compiled by our consulting scientist, Dr. LaRose, so we have every reason to believe its accuracy.

We need from you as accurate an estimate of the age of this tapestry as possible, so as to determine both whether we should pay the dealer’s usurious price and whether we should continue dealing with the company he represents. Unfortunately, we are under some time constraints, as the dealer requires an answer soon; we therefore need your report by the 22nd of this month. In addition, as it is in both of our interests to assure the successful and timely completion of your project, our consulting scientist, Dr. Gavin LaRose, is available to answer any and all additional questions that you may have regarding the technicalities or requirements related to your effort. You are urged to contact him as a group with any questions you may have—and, as is also indicated in your contract, you should see him (also as a group) to give a progress report on the project by the 15th.

Sincerely,

M. Ike Langelo, President
Art Inc.
Some Observations

we’ll look at solutions & grading in a moment…

• I like “really” applied projects.  
  • But “Reality” is in the eye of the beholder. . .

• Applied letters provide a context for students’ responses.

• These are fundamentally Writing Projects:
  • They are written, and require reading detailed (mathematical) writing.
  • They are projects demanding an open-ended, well-formulated (written) response.
  • This is (?) a Good Thing: writing to explain mathematics is a learning process, and useful skill.
Some Practical Logistical Matters

- These are **Non-Trivial**, for all concerned:
  - 1–3 per semester, in one class, is probably good.
  - $\approx 5$–10% of course grade/project is probably good.
  - $\approx 2$ (+) weeks to work is probably good. (I don’t use class-time for project work.)

- Small Classes ($< 30$ students) may be essential. Small Classes ($< 40$ students) may be essential.

- And Groups are an excellent thing
  - $\frac{36}{2} = 18$, and $\frac{36}{3} = 12$.
  - Problems can be *sufficiently difficult*. 
Some Practical Philosophical Matters

- **Propaganda** is a key part of these (and many other) teaching efforts:
  - Groups are good! Problem solving is great! Math is fun! These are real-life applications! You will do this in a future life!

- **Students can Write!**
  - ...but sample papers or writing tips may be good.

- **Students can Procrastinate!**
  - An early meeting with groups may be good (do students know their group members? have the project?)
  - A cut-off on aid (e.g., over the last weekend) may help.
Grading

- Holistic Grading

  * I AM ONE WITH THE INFINITY OF PAPERS I HAVE READ.
  * DO NOT QUESTION THE MASTER.

  ... or, point-based grading:
  
  - Rubrics
    
    * Emphasize the mathematical solution to the problem. Structure, (quality,) and format of the solution paper are graded implicitly.
  
  - Checklists
    
    * Emphasize general mathematical process and structure. The specific solution to the problem is graded implicitly.

...but is graded!
Rubrics (and how to create them)

- **Identify steps** in the solution to the problem.
  - *e.g.*, Find exponential decay model; Verify approximate age of tapestry; Calculate actual age

- **Assign points**, usually 1–4 per step
  - *e.g.*, exponential model:
    - 0 = no sensible model
    - 1 = model, error in all params
    - 2 = 1 param correct, or 2 w/poor derivation.
    - 3 = 2 param correct, or 3 w/poor derivation
    - 4 = correct and well-explained model

  (or, equivalently, keep running list of errors and classes of errors.)

- Assign some points for **clarity/quality** and **deadlines** (15–20 total)

- Read each paper **once** + spot-check.

  . . . this is the point...
## Sample Rubric

**For project *Into the World of Art***

<table>
<thead>
<tr>
<th>Objective</th>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
<th>3 points</th>
<th>4 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential model</td>
<td>None</td>
<td>Incorrect exponential model</td>
<td>Exponential model, 1 correct parameter, or 2 w/poor derivation</td>
<td>2 correct parameters, or 3 w/poor derivation</td>
<td>correct, well explained model</td>
</tr>
<tr>
<td>Correct claim</td>
<td>No valid age</td>
<td>Say &lt;2000 yr, poor argument</td>
<td>Say &lt;2000 yr, intuitive or math. argument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tapestry age</td>
<td>No valid age</td>
<td>Estimate, poor argument</td>
<td>Estimate, referring to data, but with errors or unclear argument</td>
<td>Accurate estimate, generally correct argument, not fully supported or clear</td>
<td>Accurate and well supported age estimate</td>
</tr>
<tr>
<td>Clarity &amp; Organization</td>
<td>Very bad</td>
<td>Multiple unclear sections or limited organization</td>
<td>Some unclear section(s), spotty organization</td>
<td>Good paper, few flaws</td>
<td></td>
</tr>
<tr>
<td>Deadlines</td>
<td>None met</td>
<td>One missed</td>
<td>All met</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Checklists and where to find them

- A **Checklist** indicates what is expected of a student paper, e.g.,
  - **Required sections** (problem statement, problems solution, acknowledgement . . .);
  - **Content** (explanation of solution, definitions, labeled diagrams . . .);
  - **Format** (scientific paper or business letter, spelling and grammar . . .).

- **Worth 1–2 points each**
- **Total = 10–15 points**
- **Read each paper once + spot-check.**

… sources: Elyn Rykken, Annalisa Crannell
Sample Checklist (from Elyn Rykken, Muhlenberg College)

Instructions given to students with checklist:

- Please attach this page with a paper-clip to your writing assignment.
- This list will be used to grade your assignment and will be returned to you with comments. Keep a copy of your paper for your own reference.
- Use this checklist as a guide for yourself while writing the assignment.

<table>
<thead>
<tr>
<th>Form:</th>
<th>5 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>1. Clearly (re)state the problem to be solved (including essential details)?</td>
</tr>
<tr>
<td></td>
<td>2. Explain what level and types of math will be used?</td>
</tr>
<tr>
<td></td>
<td>3. Solve the question that was originally asked? 2 pts</td>
</tr>
<tr>
<td></td>
<td>4. Give acknowledgement where it is due (did anyone work with you on the math)?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Content:</th>
<th>7 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>5. Give a precise and well-organized explanation of how the answer was found? 2 pts</td>
</tr>
<tr>
<td></td>
<td>6. Define all variables, terms and notation used?</td>
</tr>
<tr>
<td></td>
<td>7. Explain how each formula is derived or where it can be found?</td>
</tr>
<tr>
<td></td>
<td>8. Clearly label diagrams, tables, graphs or other visual representations of the math?</td>
</tr>
<tr>
<td></td>
<td>9. Contain correct mathematics? 2 pts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Presentation:</th>
<th>3 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>10. Use standard business letter form?</td>
</tr>
<tr>
<td></td>
<td>11. Use correct spelling, grammar, and punctuation?</td>
</tr>
<tr>
<td></td>
<td>12. Look neat? (Typing helps with this.)</td>
</tr>
</tbody>
</table>
A Practical Philosophy of Project Groups

• Cross-group collaboration is probably ok.
• Propaganda is a key part of these teaching efforts:
  Groups are pedagogically sound!
  Groups make hard problems easier!
  Groups are universally loved and appreciated!

• Composition:
  • Self-selected
  • Heterogeneous
  • Four, three
  • Instructor-assigned
    mix-up groups, delegate blame
  • Match ability
    pedagogy? group dynamic?
  • Two students
    pedagogy? freeloaders?
Random Other Thoughts

- It’s probably a good idea to **solve projects yourself**.
- In general, **students rise to challenges** (both in *writing* and *problem-solving*).
- **Problem topics** should fit your syllabus:
  
  Syllabus $\rightarrow$ topic $\rightarrow$ project

  e.g., Ma316, W09:

  ... so the first project has to do with *first-order DE*

- There are many good **Project Resources**
Concluding Thoughts

- There are varying levels of realism in these projects. *I like “real applications.”*

- **Grading** does *not* have to take the better part of a lifetime. *But it takes some time.*

- **Students** might (will?) *complain*, but will (might?) also like the projects.

- I’ve never stopped to teach **writing**. *But I do mark errors.*

- It may be defensibly argued that projects are **fun** and **productive**.

- It is *not* necessary to write all your own projects: **adapt**, **reuse**, **recycle**. *Others’ projects, and your own.*
Resources


- Textbooks and modeling texts, esp. Hughes-Hallett et al. (Wiley); also *Mathematical Modeling*..., Hadlock (MAA, '98); *Mathematical Biology*, Murray (Springer ’90); Differential Equations texts (esp. Borrelli & Coleman, Wiley ’04)...

- Others’ projects on-line (Annalisa, me, Tommy, Elyn...).

