Effectively Using Applied Writing Projects in Undergraduate Mathematics Courses

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Compartment Drug Model

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Overview

- Introduction
  - Definitions: what are “Applied Writing Projects”?  
  - Some Logic: and why might we use them? 
  - Logically, then An Example 
  - Some Observations and Context 

- Implementation
  - Some Logistical and Practical matters 
  - The Elephant in the room. . . 

- Groups and Other Thoughts
  - Groups and how they might be set up
  - Other Advice and Selecting Projects

- Conclusions and Resources
What an applied writing project is

- A Really Big Story Problem
  - An *Applied*, (mostly) *Real-World*,
  - *Multi-step*, often *Multi-part* problem.

- Presented as a Contract Letter (or, letter requesting help):
  - *Dear IMC: The Spate Gallery has recently been approached by an antiques dealer...*
  - *Dear Calculus Students, I have a problem...*

- Requiring a Written, Multi-page Solution (3–7 pages)
  - ...and a commensurate amount of work.
Why we might use them

- Applications are fun and motivational, & show mathematics beyond the boundary of the textbook, as having importance and relevance in its own right.
- They provide a vehicle to focus on substantial material being covered
- They promote things we want for our students:
  - conceptual understanding and problem solving.
  - Writing skills
- And there is research and analysis arguing that writing promotes learning.

...and they can be graded without instructor fatalities
Dear IMC:

The Spate Gallery has recently been approached by an antiques dealer with headquarters in the outskirts of Athens who has offered us first purchase rights to a so-called ancient tapestry. However, we are concerned with its authenticity and so are contacting you to ascertain its age.

The dealer asserts that the tapestry is 2000 years old, having been loomed sometime in the first century B.C. In order to determine the veracity of this claim, our technical analysis department was able to obtain a small portion of the tapestry and has done an analysis of the Carbon-14 present therein. They report the presence of 190 nanograms (ng) of Carbon-14 in the 1 gram sample they took.

As you will, of course, know from your own references on the subject, the amount of C14 present in the environment has varied with time. This amount is constant in living creatures, and then decays (with a half-life of 5,780 years) after they die. The figure below shows the amount (in ng) of C14 (per gram) present in the environment for the past 2,100 years. This was compiled by our consulting scientist, Dr. LaRose, so we have every reason to believe its accuracy.

We need from you as accurate an estimate of the age of this tapestry as possible, so as to determine both whether we should pay the dealer’s usurious price and whether we should continue dealing with the company he represents. Unfortunately, we are under some time constraints, as the dealer requires an answer soon; we therefore need your report by the 22nd of this month. In addition, as it is in both of our interests to assure the successful and timely completion of your project, our consulting scientist, Dr. Gavin LaRose, is available to answer any and all additional questions that you may have regarding the technicalities or requirements related to your effort. You are urged to contact him as a group with any questions you may have—and, as is also indicated in your contract, you should see him (also as a group) to give a progress report on the project by the 15th.

Sincerely,

M. Ike Langelo, President
Art Inc.
Some Observations and Context

we’ll look at solutions & grading in a moment... 

• I like “really” applied projects.
  • But “Reality” is in the eye of the beholder...

• The letter format provides a context for students’ responses.

• These are fundamentally, and intentionally, Writing Projects:
  • They require reading and writing detailed (mathematical) prose.
  • They are projects demanding problem-solving and multi-stage solutions.

• They can be used in many classes:
  • Calculus I, II, & III; Linear Algebra; Differential Equations...
Some Practical Logistical Matters

- These are **Non-Trivial**, for all concerned:
  - 1–3 per semester, in one class, is probably good.
  - \( \approx 5–10\% \) of course grade/project is probably good.
  - \( \approx 2 (+) \) weeks to work is probably good. (I don’t use class-time for project work.)

- **Small Classes** (<40 students) may be essential.

- And **Groups** are an excellent thing
  - \( \frac{36}{2} = 18 \), and \( \frac{36}{3} = 12 \).
  - Problems can be **sufficiently difficult**.
Some Practical Philosophical Matters

- **Enthusiastic promotion** is a key part of these (and many other) teaching efforts:
  - Groups are good! Problem solving is great! Math is fun! These are real-life applications! You will do this in a future life!

- Students **can Write**!
  - …but sample papers or writing tips may be good.

- Students **can Procrastinate**!
  - An early meeting with groups may be good
    (do students know their group members? have the project?)
  - A cut-off on aid (e.g., over the last weekend) may help. or may not.
- **Holistic Grading**
  
  I am one with the infinity of papers I have read.
  
  Do not question the master.

... or, *point-based grading*:

- **Rubrics**
  
  - Emphasize the *mathematical solution* to the problem.
    
    *Structure, (quality,) and format* of the solution paper are graded *implicitly*.

- **Checklists**
  
  - Emphasize *general mathematical process* and *paper structure*.
    
    The *specific solution* to the problem is graded *implicitly*.
Rubrics

- **Identify steps** in the solution to the problem.
  - *e.g.*, Find exponential decay model; Verify approximate age of tapestry; Calculate actual age

- **Assign points**, usually 1–4 per step
  - *e.g.*, exponential model:
    - 0 = no sensible model
    - 1 = model, error in all params
    - 2 = 1 param correct, or 2 w/poor derivation.
    - 3 = 2 param correct, or 3 w/poor derivation
    - 4 = correct and well-explained model

  (or, equivalently, assign points and keep running list of errors and classes of errors with their penalties.)

- **Assign some points for clarity/quality and deadlines** (15–20 total)

- **Read each paper once + spot-check.**
Sample Rubric

For project *Into the World of Art*

<table>
<thead>
<tr>
<th>Objective</th>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
<th>3 points</th>
<th>4 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential model</td>
<td>None</td>
<td>Incorrect exponential model</td>
<td>Exponential model, 1 correct parameter, or 2 w/poor derivation</td>
<td>2 correct parameters, or 3 w/poor derivation</td>
<td>correct, well explained model</td>
</tr>
<tr>
<td>Correct claim</td>
<td>No valid age</td>
<td>Say &lt;2000 yr, poor argument</td>
<td>Say &lt;2000 yr, intuitive or math. argument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tapestry age</td>
<td>No valid age</td>
<td>Estimate, poor argument</td>
<td>Estimate, referring to data, but with errors or unclear argument</td>
<td>Accurate estimate, generally correct argument, not fully supported or clear</td>
<td>Accurate and well supported estimate</td>
</tr>
<tr>
<td>Clarity &amp; Organization</td>
<td>Very bad</td>
<td>Multiple unclear sections or limited organization</td>
<td>Some unclear section(s), spotty organization</td>
<td>Good paper, few flaws</td>
<td></td>
</tr>
<tr>
<td>Deadlines</td>
<td>None met</td>
<td>One missed</td>
<td>All met</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Observation:* This must be distributed after students have finished the project.
Checklists

- A Checklist indicates what is expected of a student paper, e.g.,
  - Required sections (problem statement, problems solution, acknowledgement...);
  - Content (explanation of solution, definitions, labeled diagrams...);
  - Format (scientific paper or business letter, spelling and grammar...).

- Worth 1–2 points each
- Total = 10–15 points
- Read each paper once + spot-check.

...sources: Elyn Rykken, Annalisa Crannell
Sample Checklist  (from Elyn Rykken, Muhlenberg College)

Instructions given to students with checklist:

- Please attach this page with a paper-clip to your writing assignment.
- This list will be used to grade your assignment and will be returned to you with comments. Keep a copy of your paper for your own reference.
- Use this checklist as a guide for yourself while writing the assignment.

<table>
<thead>
<tr>
<th>Form:</th>
<th>5 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>1. Clearly (re)state the problem to be solved (including essential details)?</td>
</tr>
<tr>
<td></td>
<td>2. Explain what level and types of math will be used?</td>
</tr>
<tr>
<td></td>
<td>3. Solve the question that was originally asked? 2 pts</td>
</tr>
<tr>
<td></td>
<td>4. Give acknowledgement where it is due (did anyone work with you on the math)?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Content:</th>
<th>7 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>5. Give a precise and well-organized explanation of how the answer was found? 2 pts</td>
</tr>
<tr>
<td></td>
<td>6. Define all variables, terms and notation used?</td>
</tr>
<tr>
<td></td>
<td>7. Explain how each formula is derived or where it can be found?</td>
</tr>
<tr>
<td></td>
<td>8. Clearly label diagrams, tables, graphs or other visual representations of the math?</td>
</tr>
<tr>
<td></td>
<td>9. Contain correct mathematics? 2 pts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Presentation:</th>
<th>3 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>10. Use standard business letter form?</td>
</tr>
<tr>
<td></td>
<td>11. Use correct spelling, grammar, and punctuation?</td>
</tr>
<tr>
<td></td>
<td>12. Look neat? (Typing helps with this.)</td>
</tr>
</tbody>
</table>
Some thoughts about groups

- Cross-group collaboration is probably ok.
- Enthusiastic promotion is a key part of these teaching efforts:
  Groups are *pedagogically sound!*
  Groups make *hard problems easier!*
  Groups are *universally loved and appreciated!*
- Composition:
  - Self-selected
  - Heterogeneous
  - Four, three
  - Instructor-assigned
    - mix-up groups, delegate blame
  - Match work ethic
    - pedagogy? group dynamic?
  - Two students
    - pedagogy? freeloaders?

... standard disclaimers apply
Other thoughts, partially at random

- It’s probably a good idea to **solve projects yourself**.
- In general, **students rise to challenges** (both *writing* and *problem-solving*).
- **Problem topics** should fit your syllabus & goals:
  
  Syllabus → topic → project

  *e.g., Ma316, F10:*

  …so the first project has to do with *first-order DE*

- There are many good **Project Resources**
Concluding Thoughts

- There are varying levels of realism in these projects. I like “real applications.”
- Grading does not have to take the better part of a lifetime. But it takes some time.
- Students might (will?) complain, but will (might?) also like the projects.
- I’ve never stopped to teach writing. But I do mark errors.
- It may be defensibly argued that projects are fun and productive.
- It is not necessary to write all your own projects: adapt, reuse, recycle. Others’ projects, and your own.
Resources

- Textbooks and modeling texts, esp. Hughes-Hallett et al. (Wiley); also *Mathematical Modeling...*, Hadlock (MAA, ’98); *Mathematical Biology*, Murray (Springer ’90); Differential Equations texts (esp. Borrelli & Coleman, Wiley ’04). . .
- CODEE (for DiffEq: <http://www.codee.org/>).
- Others’ projects on-line (Annalisa, me, Tommy, Elyn...).