

6 Essential Questions for Problem Solving

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6 ESSENTIAL QUESTIONS FOR PROBLEM SOLVING

Martina joined my precalculus class in November of her senior year of high school. (The name and some details of this student's experience have been changed to protect her identity.) She told me that she did not consider herself to be good at math, but she felt that a good grade in precalculus would be an important part of her college applications. She also told me that college was critical to her ability to live a better life than she had had as a child and that she was certain she would be able to do well in precalculus as long as I was very clear about exactly what I wanted her to do.

One of the primary expectations I have for my students is for them to develop greater independence when solving complex and unique mathematical problems. Martina joined the majority of the class in telling me that she was comfortable with explicit, step-by-step instructions, but that working independently on math problems that were different from those she had previously been shown how to solve was very difficult. Martina was especially emphatic about the nature and degree of this challenge. The expectations that many of my students had of me with regard to prescriptive, step-by-step instructions were at odds with my expectations for them—and yet we avoided conflict. The story of how I supported my students as they gained confidence and independence with complex and unique problem-solving tasks, while honoring their expectations with regard to clear, explicit instruction, is rooted in a set of guiding questions I call *essential questions for problem solving*.

Increase students' access to and success with complex quadratic function tasks in any middle school, high school, or early college mathematics class.

Nancy Emerson Kress

WHY WE USE ESSENTIAL QUESTIONS FOR PROBLEM SOLVING

One challenge that teachers face is developing instructional methods that support the continued growth of successful problem solvers and simultaneously nurturing the development of confidence and enabling success among students who struggle. It is incumbent on the mathematics teaching community to implement teaching strategies that genuinely reflect the belief that all students are able to learn and do mathematics (Boaler 2016; Dweck 2007) and that provide every student with robust support and rich opportunities to expand their problem-solving skill set.

A strategy commonly used to improve students' success with problem solving is to increase their exposure to challenging and interesting problems. Providing support and teaching students that the key to success is perseverance (Boaler 2016; Dweck 2007) may increase their ability to solve a variety of problems in the future. However, experience and perseverance alone do not consistently lead all students to become successful with complex or unique problem-solving tasks.

Students are not all equally prepared to participate in open curricular and reform approaches to learning mathematics (Boaler 2002). There is concern that some students may be less aware of the particular mathematical practices that are being used and developed in their classes (Ball et al. 2005; Boaler 2002; Lubienski 2000; Selling 2016), and for many this is because they expect teaching to be more direct (Delpit 1988). This concern is especially strong in relation to students from lower socioeconomic status or working-class backgrounds, students who speak English as a second language, and students who belong to minority racial or ethnic groups (Ball et al. 2005; Boaler 2002; Delpit 1988; Lubienski 2000; Parks 2010).

The term *explicit*, as applied to mathematics teaching, is often associated with step-by-step procedural instruction. This is the form of instruction my students were most comfortable with. But Boaler (2002) cautions against responding to equity concerns with a return to more direct teaching methods because direct instruction can reduce students' opportunities to engage in sense making about complex problems (Boaler 2002; Greeno and Boaler 2000; Schoenfeld 1992; Selling 2016).

Improving equity, particularly for students who respond positively to direct teaching methods, without resorting to prescriptive methods of teaching requires a more nuanced understanding of what it means to be explicit. Selling (2016) suggests an alternative to being explicit that centers on bringing direct attention to mathematical practices being used in the classroom. She claims that “participants

EXPERIENCE AND PERSEVERANCE ALONE DO NOT CONSISTENTLY LEAD ALL STUDENTS TO BECOME SUCCESSFUL WITH COMPLEX OR UNIQUE PROBLEM-SOLVING TASKS.

in this interaction may be more or less aware that they (or others) are engaging in particular mathematical practices” (p. 510). She suggests that a form of being explicit that is direct about highlighting mathematical practices, as opposed to step-by-step instructions, enables all students to be equally aware of the strategies and practices being used. This interpretation has important implications for increasing equitable access to mathematics for students from widely varied backgrounds.

ESSENTIAL QUESTIONS AS A FRAMEWORK

The essential questions described in this article are designed to provide a framework to support students to pose purposeful questions (NCTM 2014, p. 35) about complex mathematical problems, and they are consistent with design principles for active learning (Webb 2016). Although these questions differ significantly from the questions and suggestions that Pólya proposed (1945), they take a similar approach in the sense that they are applicable to a wide range of problem-solving tasks and do not prescribe specific mathematical steps for solving a particular type of problem. I used these questions in second-year algebra and precalculus classes to support increased learning opportunities for all students.

The first step in developing essential questions for problem solving was to identify a specific skill set that would support reliable and consistent success at problem solving for all students. It involved observing precisely which actions students were taking that lead to success, regardless of whether students themselves consciously identified the critical practices they were using. The actions and skills identified as both fundamental and comprehensive for supporting problem-solving success across a wide range of problem types are as follows:

- Noticing, or making observations
- Asking questions
- Knowing how and why to carry out particular mathematical actions
- Verifying accuracy

Desirable strategies that are less frequently applied, even among highly successful students include these:

- Making connections
- Extending the problem

A study of the alignment of the questions to the Common Core's (CCSSI 2010) Standards for Mathematical Practice (SMP) was carried out (Kress 2014), and the questions were refined over the course of two years of application in precalculus and second-year algebra.

HOW TO INTRODUCE SIX ESSENTIAL QUESTIONS FOR PROBLEM SOLVING

When I first introduce these questions to a class, I use a prompt—just a single quadratic function—that is not complex or unique at all. The task's lack of complexity allows me to introduce the questions as a structure to support students' exploration of mathematics. They can make observations, ask for additional information, and try out ideas. After one fifty-minute class period, students see how previously isolated topics fit together to form a big picture. Students seem to gain satisfaction from the experience of making multiple connections between concepts that they have previously experienced as isolated, and use of the questions increases both the depth and breadth of students' understanding of quadratic functions and their graphs.

The six questions are listed below, followed by a description of the purpose and role of each question, as well as examples of how my students took up and responded to the questions.

1. What do you notice?
2. What additional information or clarification would be helpful?

3. What can you do or figure out?
4. How do you know that your work and/or answer are accurate?
5. Is there another way you could approach this problem?
6. What else can you say about the problem, and what else would you like to know?

WHAT DO YOU NOTICE?

Many students immediately attempt to begin mathematical work on problems without pausing to consider the overall picture or subtle details. This first question supports students in taking stock of what they know before they get embroiled in the complexities of the task. This question also provides the teacher with formative assessment information.

When Martina and her precalculus classmates were asked to consider a single quadratic function as a prompt and were asked, "What do you notice?" the responses included observations such as these:

- "There's a little two above the x ."
- "It's a quadratic."
- "You could factor."
- "There are three terms."
- "You could graph it."
- "I think the graph might be a parabola."

Second-year algebra students who had been introduced to quadratic functions in their first algebra course responded similarly. I made certain that every student response was accepted and publicly recorded.

WHAT ADDITIONAL INFORMATION OR CLARIFICATION WOULD BE HELPFUL?

Some students hesitate to ask questions. Others ask for help without putting effort into refining their questions or identifying what aspect of the problem requires clarification. This prompt legitimizes students asking for additional information while supporting student ownership and responsibility for the thought process.

When I asked students to consider a quadratic function and, "What additional information or clarification would be helpful?" they responded with questions such as the following:

- "What does the two above the x mean?"
- "How would you factor that?"
- "What does the graph look like?"
- "How do you know it's a quadratic?"

Questions can be answered immediately, either by student volunteers or by the teacher. If they are not answered immediately, then establishing a strategy to ensure that all questions are answered in the course of that class period is imperative.

WHAT CAN YOU DO TO FIGURE IT OUT?

This is the point at which students do the work of attempting to determine an answer if the prompt calls for one. In the context of a quadratic function prompt, my students typically explore the mathematics and draw connections and conclusions.

In second-year algebra and precalculus, my students did some or all of the following:

- Factored
- Made a table of values
- Drew a graph
- Solved for x -intercepts using the quadratic formula
- Stated x -intercepts
- Stated the y -intercept
- Stated the coordinates of the vertex

Letting students determine the direction of the discussion is important. Students usually think of new ideas that build off one another's responses, but the order in which this work happens will vary from class to class. I facilitate the discussion by calling on students and taking detailed notes on the board. Another teacher might opt to ask a student to take the notes on the board. It is important that the notes are visible to everyone as the students work together. By the end of this stage, my classes have created a board covered with mathematical calculations and information related to the quadratic function on which they are working.

HOW DO YOU KNOW THAT YOUR WORK AND ANSWER ARE ACCURATE?

Students have many different methods of verifying accuracy. One is to solve the problem in a different way, confirming that the same result is obtained. Students may also consider whether their observations contain contradictions, substitute an answer back into an equation to verify its validity, or check that the solution obtained from an equation agrees with what is shown on a graph. Methods vary greatly by student as well as by type of problem.

Martina and her classmates, when applying this question to the quadratic function $f(x) = x^2 + 3x - 4$, responded in the following ways:

- “The y -intercept on the graph is at -4 , and in the equation, when you make $x = 0$, then $y = -4$.”
- “The x -intercepts are at 1 and -4 , and the factors are $(x + 4)$ and $(x - 1)$.”
- “The graph opens up, and the coefficient of x^2 is positive.”
- “The graph is symmetrical, so it looks right.”

If some of their work contains an error, they may observe the following:

- “We drew the graph with a y -intercept at -3 , but the constant in the equation is -4 . That doesn't make sense.”
- “Our graph crosses the x -axis at 4 , but when I substitute 4 into the function, I don't get 0 for an answer.”

If students are slow to respond, follow-up questions are necessary. I used such probing questions as these:

- “How are the y -intercept and constant in the function related to each other?”
- “What do you notice if you substitute 4 into the function for x ?”

IS THERE ANOTHER WAY YOU COULD APPROACH THIS PROBLEM?

Students often address this question in conjunction with the previous one because solving the problem in another way is an efficient method of confirming the accuracy of their work. They are presented separately because both topics are of significant importance. Because they can be answered separately, considering them as unique concerns is valuable.

This consideration does not always result in a second practical method of solving the problem, but even when it does not, students will typically gain additional insights through investigating the possibility of other angles.

When considering a quadratic function, this is the point at which my students are likely to notice if they have omitted the use of a familiar strategy, such as the quadratic formula or factoring. They add that work to what they have done previously, further strengthening their ability to confirm that their previous observations make sense.

WHAT ELSE CAN YOU SAY ABOUT THE PROBLEM, AND WHAT ELSE WOULD YOU LIKE TO KNOW?

These questions serve the purpose of prompting students to stop and think before moving on. Having arrived at a solution and confirmed that the process and answer are accurate, students are frequently prepared to be finished with a task they feel has been completed. This question encourages students to reflect on the ways in which the work fits into their larger experience or general knowledge base. It also provides additional assessment information to the teacher.

Students in my second-year algebra and precalculus classes asked questions and made new observations such as these:



TEACHING STUDENTS TO USE THESE ESSENTIAL QUESTIONS PURPOSEFULLY AS PROMPTS TO WORK THROUGH STAGES OF SOLVING PROBLEMS INCREASED ENGAGEMENT.

- “What would make the graph open down?”
- “Can a parabola open sideways?”
- “What would the graph look like if there were a different coefficient for x^2 ?”
- “The $+/-$ in the quadratic formula gives you values to the right and left of the vertex, or the axis of symmetry. That’s how you get the x -intercepts. You could even write it as this:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .”$$

Teaching students to use these essential questions purposefully as prompts to work through stages of solving problems increased engagement in my classes. The questions shaped and directed students’ thinking and supported all students in becoming aware of the use of these practices for solving mathematical problems. Students’ confidence increased, and they began doing work that was more thorough and complete.

NEXT STEPS: MOVING TOWARD INDEPENDENCE

When Martina first joined my class, she struggled to participate within the existing norms of the classroom. When the class engaged in discourse about mathematics, collective sense making around open-ended problems, or communal exploration of multiple methods of solution, she did not participate. If I did not provide immediate explanations of step-by-step procedures for solving problems, she got frustrated and stopped participating for the rest of the class period.

The structure of the essential questions for problem solving supported Martina to reduce her dependence on procedural instruction. Her willingness to engage in open-ended and complex tasks gradually increased, and she became more likely to participate in class and group discussions. She developed the ability to generate observations, ideas, and solution strategies independently. She and her classmates built on their experience working with a simple quadratic function prompt, and they became more comfortable working on complex problems such as the following:

To celebrate the Fourth of July, a city has hired Star Burst, Inc. to launch fireworks into the air from the top of a tower 20 feet tall. The fireworks can be fired with an initial upward velocity of 128 feet per second. Write a mathematical model for this scenario, and use your model to find how many seconds after launch the fireworks attains its maximum height (assuming it has not yet exploded). What is its height above ground at this time? Explain how you know that this is the maximum height. If you wanted the fireworks to reach its maximum height exactly 5 seconds after launching, how might you accomplish this?

Early in the school year, my students would have responded to the problem described above by asking for demonstration of step-by-step strategies using a nearly identical problem. Later in the year, Martina and her classmates were able to read such a problem and use the essential questions to work their way through the scenario. They developed greater ability to engage in productive struggle (NCTM 2014, pp. 48–52), and they demonstrated the ability to persist through a process to solve problems unlike those they had seen before.

I found that the essential questions proved useful with a variety of concepts and problem types, including—but not limited to—graphing rational, higher-order polynomial, trigonometric, exponential, and logarithmic functions as well as a variety of modeling scenarios.

IN CONCLUSION

Teaching problem-solving skills equitably is challenging. Detailed procedural instruction supports students in the short term but runs the risk of undermining students' independence. Being explicit about mathematical practices that lead to effective problem solving has the potential to increase equitable access to high-level learning about complex mathematical problem-solving tasks. Essential questions for problem solving differentiate and personalize instruction by providing structure for students who need it, while helping successful students to recognize and take ownership of the actions underlying their success.

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