MATH 289, PROBLEM SET 1
DUE: 9/8/2004

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Hand in solutions to 4 problems from the following list of problems: **Larson**, **1.1.6******, **1.1.8*****, **1.1.12*****, **1.2.5****, **1.2.7*****, **1.3.8****, **1.3.13*****, **1.3.15****. You may also choose from the following problems:

(1) ***** Suppose you have two unusual dice. The first die has positive integers $a_1 \leq a_2 \leq \cdots \leq a_6$ on its sides, and the second die has positive integers $b_1 \leq b_2 \leq \cdots \leq b_6$. If both dice are rolled, then the distribution of the outcome is exactly the same as for two regular dice. For example the probability that one throws 4 is $\frac{3}{36}$ etc. (for regular dice there are three possibilities to get 4, namely $1+3$, $2+2$ and $3+1$). If these new dice are not regular dice, what positive integers are on the side of each die. (There is essentially only one other solution!)

(2) ** Give a formula for the sequence S_n with $S_0 = 0$, $S_1 = 1$, $S_2 = 4$, $S_{n+1} = 3S_n - 3S_{n-1} + S_{n-2}$.

(3) ***** (IMO 1983) Let a, b, c be the lengths of the sides of a triangle. Prove that

$$a^2b(a-b) + b^2c(b-c) +^2 a(c-a) \geq 0.$$

Determine when equality occurs.

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