MATH 289, FALL 2005, PROBLEM SET 1

DUE: 9/14/2005

Hand in solutions for 3 of the problems below.

Problem 1. Let $a_1, a_2, \ldots, a_n$ represent an arbitrary arrangement of the numbers $1, 2, \ldots, n$. Prove that, if n is odd, the product

$$(a_1 - 1)(a_2 - 2) \cdots (a_n - n)$$

is an even number.

Problem 2. Prove that

$$\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} = \sqrt{6}.$$

Problem 3. Show that there are no positive integers x, y such that $y^2 = x^2 + x + 1$.

Problem 4. Let a, b be nonzero numbers with

$$a + b = \frac{1}{a} + \frac{1}{b}.$$

Prove that

$$a^3 + b^3 = \frac{1}{a^3} + \frac{1}{b^3}.$$

Problem 5. Let $f(x)$ be a polynomial of degree n with real coefficients and such that $f(x) \geq 0$ for every real number x. Show that

$$f(x) + f'(x) + f''(x) + \cdots + f^{(n)}(x) \geq 0$$

for all real x. ($f^{(k)}(x)$ is the k-th derivative.)

Problem 6. Let S_n be the set of all pairs (x, y) with integral coordinates such that $x \geq 0$, $y \geq 0$ and $x + y \leq n$. Show that S_n cannot be covered by the union of n straight lines.