# HOMEWORK 4, MATH 425, SECTION 3

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**Exercise 1** (Ch 4, problems, 7).

(a) $\{1,2,3,4,5,6\}$
(b) $\{1,2,3,4,5,6\}$
(c) $\{2,3,4,5,6,7,8,9,10,11,12\}$
(d) $\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$.

**Exercise 2** (Ch 4, problems, 8).

(a)

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(i)$ | $\frac{1}{36}$ | $\frac{3}{36}$ | $\frac{5}{36}$ | $\frac{7}{36}$ | $\frac{9}{36}$ | $\frac{11}{36}$ |

(b)

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(i)$ | $\frac{11}{36}$ | $\frac{9}{36}$ | $\frac{7}{36}$ | $\frac{5}{36}$ | $\frac{3}{36}$ | $\frac{1}{36}$ |

(c)

| $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(i)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

(d)

| $i$ | $-5$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(i)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

**Exercise 3** (Ch 4, problems 21).

(a) $EX > EY$: If you are choosing an arbitrary student than the probability to choose the bus with the largest number of student is $> \frac{1}{4}$, where if you choose a random driver than each bus is equally likely. So in $EX$, busses with a lot of students are weighted more.

(b)

$$EX = \frac{40}{148} \cdot 40 + \frac{33}{148} \cdot 33 + \frac{25}{148} \cdot 25 + 50 \cdot \frac{50}{148} = \frac{2907}{74} \approx 39.2838$$

$$EY = \frac{1}{4} \cdot 40 + \frac{1}{4} \cdot 33 + \frac{1}{4} \cdot 25 + \frac{1}{4} \cdot 50 = \frac{148}{4} = 37.$$

**Exercise 4** (Ch 4, problems, 24). Let $X$ be the amount that $B$ wins. If $A$ writes a 1, then

$$EX = p \cdot 1 + (1-p)(-\tfrac{3}{4}) = -\tfrac{3}{4} + \tfrac{7}{4}p.$$

If $A$ writes a 2, then

$$EX = p \cdot (-\tfrac{3}{4}) + 2 \cdot (1-p) = 2 - \tfrac{11}{4}p.$$

By drawing the graphs, it is clear that

$$\min\{-\frac{3}{4} + \frac{7}{4}p, 2 - \frac{11}{4}p\}$$

is maximal when

$$-\frac{3}{4} + \frac{7}{4}p = 2 - \frac{11}{4}p$$

It follows that

$$\frac{18}{4}p = \frac{11}{4}$$

and $p = \frac{11}{18}$. In that case

$$EX = -\frac{3}{4} + \frac{7}{4} \cdot \frac{11}{18} = \frac{23}{72}.$$

**second part:** If $B$ chooses 1, then

$$EX = q \cdot 1 + (1-q)(-\tfrac{3}{4}) = -\tfrac{3}{4} + \tfrac{7}{4}q$$

If $B$ chooses 2 then

$$EX = q \cdot (-\tfrac{3}{4}) + 2 \cdot (1-q) = 2 - \tfrac{11}{4}q.$$

Now $B$ tries to minimalize

$$\max\{\frac{3}{4} + \frac{7}{4}q, 2 - \frac{11}{4}q\}$$

Again, this maximum is where

$$\frac{3}{4} + \frac{7}{4}q = 2 - \frac{11}{4}q$$

so

$$q = \frac{11}{18}$$

The expected loss of $B$ is in that case $\frac{23}{72}$.

**Exercise 5** (Ch 4, problems 35)**.** Let $X$ be the amount that you win.

$$P(X = 1.1) = \frac{2 \cdot \binom{5}{2}}{\binom{10}{2}} = \frac{20}{45} = \frac{4}{9}.$$

and

$$P(X = -1) = 1 - \frac{4}{9} = \frac{5}{9}.$$

Hence

$$EX = \frac{4}{9}(1.1) - \frac{5}{9} \cdot 1 = -\frac{1}{15} \approx -0.06667$$

Also

$$E(X^2) = \frac{4}{9} \cdot (1.1)^2 + \frac{5}{9} \cdot 1^2 = \frac{82}{75} \approx 1.0933$$

and

$$\mathrm{Var}(X) = E(X^2) - E(X) = \frac{82}{7} - \big(\frac{1}{15}\big)^2 = \frac{49}{45} \approx 1.089$$

**Exercise 6** (Ch 4, theoretical exercises, 6)**.**

$$\sum_{i=1}^{\infty} P(N \geq i) = \sum_{i=1}^{\infty} \sum_{j=i}^{\infty} P(N = j) = \sum_{1 \leq i \leq j} P(N = j) =$$

$$\sum_{j=1}^{\infty} \sum_{i=1}^{j} P(N = j) = \sum_{j=1}^{\infty} jP(N = j) = \sum_{j=0^{\infty}} jP(N = j) = EN.$$