Exercise 1 (Ch 5, problems, 1). (a)

\[ 1 = \int_{-\infty}^{\infty} f(x) \, dx = \int_{-1}^{1} c(1-x^2) \, dx = c(x-x^3/3)|_{-1}^{1} = c((1-1/3)-(1+1/3)) = c(4/3) \]

so \( c = 3/4 \).

(b) \( F(x) = 0 \) for \( x < -1 \) and \( F(x) = 1 \) for \( x > 1 \). If \( -1 < x < 1 \) then

\[ F(x) = \int_{-1}^{x} \frac{3}{4}(y-y^3) \, dy = \frac{3}{4}y - \frac{1}{4}y^3|_{-1}^{x} = \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2} \]

Exercise 2 (Ch 5, problems, 4). (a)

\[ P(X > 20) = \int_{20}^{\infty} \frac{10}{x^2} \, dx = \left[ -\frac{10}{x} \right]_{20}^{\infty} = \frac{1}{2} \]

(b) \( F(x) = 0 \) for \( x < 10 \) and

\[ F(x) = \int_{10}^{x} \frac{10}{y^2} \, dy = -\frac{10}{y}|_{10}^{x} = 1 - \frac{10}{x} \]

for \( x \geq 10 \).

(c) \( P(X > 15) = 1 - F(15) = \frac{10}{15} = \frac{2}{3} \). Now we have a binomial distribution with parameter \( p = \frac{2}{3} \). The answer is

\[ \binom{6}{3} \left( \frac{2}{3} \right)^3 \left( \frac{1}{3} \right)^3 + \binom{6}{4} \left( \frac{2}{3} \right)^4 \left( \frac{1}{3} \right)^2 + \binom{6}{5} \left( \frac{2}{3} \right)^5 \left( \frac{1}{3} \right) + \binom{6}{6} \left( \frac{2}{3} \right)^6. \]

Exercise 3 (Ch 5, problems, 8). The expected lifetime is

\[ \int_{-\infty}^{\infty} xf(x) \, dx = \int_{0}^{\infty} x^2 e^{-x} \, dx = \Gamma(3) = 2! = 2 \]

(If you don’t want to use the Gamma function, which were not yet introduced in Ch 5, you have to integrate by parts twice.)

Exercise 4 (Ch 5, problems, 10). (a) Let \( X \) be amount of minutes of arrival time after 7am.

\[ P(\text{goes to } A) = P(X \in [5,15] \cup [20,30] \cup [35,45] \cup [50,60]) = \frac{10+10+10+10}{60} = \frac{2}{3}. \]

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(b)

\[ P(\text{goes to } A) = P(X \in [10, 15] \cup [20, 30] \cup [35, 45] \cup [50, 60] \cup [65, 70]) = \frac{5 + 10 + 10 + 5}{60} = \frac{2}{3}. \]

**Exercise 5** (Ch 5, theoretical exercises, 1). Integration by parts and substitution \( x = y/\sqrt{2b} \) yields:

\[
1 = \int_0^\infty ax^2 e^{-bx^2} \, dx = \left[ -\frac{a}{b} xe^{-bx^2} \right]_0^\infty + \int_0^\infty \frac{a}{b} e^{-bx^2} \, dx = \frac{a}{2b} \int_{-\infty}^\infty e^{-bx^2} \, dx = \frac{a}{2b} \sqrt{\frac{\pi}{2b}} = \frac{a\sqrt{\pi}}{2b^{3/2}}
\]

Hence \( a = \frac{2}{\sqrt{\pi}} b^{3/2} \).