HOMEWORK 6, MATH 425, SECTION 3

HARM DERKSEN

Exercise 1 (Ch 5, problems, 1). **(a)**

$$1 = int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{1} c(1-x^2)\,dx = c(x-x^3/3)]_{-1}^{1} = c((1-1/3)-(-1+1/3)) = c(4/3)$$

so $c = 3/4$.

(b) $F(x) = 0$ for $x < -1$ and $F(x) = 1$ for $x > 1$. If $-1 < x < 1$ then

$$F(x) = \int_{-1}^{x} \frac{3}{4}(y - y^2)\,dy = [\frac{3}{4}y - \frac{1}{4}y^3]_{-1}^{x} = \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2}.$$

Exercise 2 (Ch 5, problems, 4). **(a)**

$$P(X > 20) = \int_{20}^{\infty} \frac{10}{x^2}\,dx = \frac{-10}{x}]^{\infty}20 = \frac{1}{2}.$$

(b) $F(x) = 0$ for $x < 10$ and

$$F(x) = \int_{10}^{x} \frac{10}{y^2}\,dy = \frac{-10}{y}]_{10}^{x} = 1 - \frac{10}{x}$$

for $x \geq 10$.

(c) $P(X > 15) = 1 - F(15) = \frac{10}{15} = \frac{2}{3}$. Now we have a binomial distribution with parameter $p = \frac{2}{3}$. The answer is

$$\binom{6}{3}(\frac{2}{3})^3(\frac{1}{3})^3 + \binom{6}{4}(\frac{2}{3})^4(\frac{1}{3})^2 + \binom{6}{5}(\frac{2}{3})^5(\frac{1}{3}) + \binom{6}{6}(\frac{2}{3})^6.$$

Exercise 3 (Ch 5, problems, 8). The expected lifetime is

$$\int_{-\infty}^{\infty} xf(x)\,dx = \int_{0}^{\infty} x^2 e^{-x}\,dx = \Gamma(3) = 2! = 2.$$

(If you don't want to use the Gamma function, which were not yet introduced in Ch 5, you have to integrate by parts twice.)

Exercise 4 (Ch 5, problems, 10). **(a)** Let X be amount of minutes of arrival time after 7am.

$$P(\text{goes to } A) = P(X \in [5,15] \cup [20,30] \cup [35,45] \cup [50,60]) = \frac{10+10+10+10}{60} = \frac{2}{3}.$$

(b)

$$P(\text{goes to } A) = P(X \in [10,15]\cup[20,30]\cup[35,45]\cup[50,60]\cup[65,70]) = \frac{5+10+10+10+5}{60} = \frac{2}{3}.$$

Exercise 5 (Ch 5, theoretical exercises, 1)**.** Integration by parts and substitution $x = y/sqrt2b$ yields:

$$1 = \int_0^\infty ax^2 e^{-bx^2}\,dx = \left[\frac{-a}{b} x e^{-bx^2}\right]_0^\infty + \int_0^\infty \frac{a}{b} e^{-bx^2}\,dx == \frac{a}{2b}\int_{-\infty}^\infty e^{-bx^2}\,dx =$$

$$= \frac{a}{2b\sqrt{2b}}\int_{-\infty}^\infty e^{-y^2/2}\,dy = \frac{a}{2b\sqrt{2b}}\sqrt{2\pi} = \frac{a\sqrt{\pi}}{2b^{3/2}}$$

Hence $a = \frac{2}{\sqrt{\pi}} b^{3/2}$.