HOMEWORK 7, MATH 425, SECTION 3

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Exercise 1 (CH5, problem 16).

\[ p = P(X > 50) = P(Z > (50-40)/4) = P(Z > 2.5) = 1 - P(Z < 2.5) = 1 - 0.9938 = 0.0042. \]

is the probability that more than 50 inches of rain falls in a certain year. We assume that this probability is the same for all years, and that the amount of rainfall for all the years are independent of each other. Let \( X \) be the number of years until we have more than 50 inches of rainfall. This is a geometric distribution with parameter \( p \). We have

\[ P(X > 10) = (1 - p)^{10} = (0.9938)^{10} = 0.9397. \]

Exercise 2 (Ch 5, problem 28).

\[ 0.2 = P(X > 9) = P(Z > (9 - 5)/\sigma) = 1 - \Phi(4/\text{sigma}). \]

So we have \( \phi(4/\text{sigma}) = 0.8 \) and (using the table) \( 4/\text{sigma} \approx 0.84 \). Therefore \( \sigma = 4/(0.84) = 4.76 \) and \( \text{Var}(X) = \sigma^2 = 22.68 \).

Exercise 3 (Ch 5, problem 28). We assume a binomial distribution with \( p = 0.12 \) and \( n = 200 \). We approximate with a normal distribution. \( E(X) = np = 24 \) and \( \text{Var}(X) = np(1-p) = 21.12 \), \( \text{SD}(X) = \sqrt{21.12} = 4.596 \).

\[ P(X \geq 20) = P(Z > (19.5 - 24)/4.596) = 1 - \Phi(-0.98) = \Phi(0.98) = 0.8365. \]

Exercise 4 (CH5, problem 33). In view of formula (5.1) on page 212 and the discussion there, it really does not matter how old the radio was when it was bought. We might as well assume that it was brand new. So we have

\[ P(X > 8) = \int_8^\infty 8 e^{-t/8} dt = -e^{-t/8}|_8^\infty = e^{-1}. \]

Exercise 5 (Ch 5, problem 36). If \( \lambda(t) = t^3 \), then

\[ F(t) = 1 - \exp(- \int_0^t \lambda(s) ds) = 1 - \exp(\int_0^t s^3 dx) = 1 - e^{-t^4/4} \]

for \( t \geq 0 \).

(a) \( P(X > 2) = 1 - F(2) = e^{-4} = 0.0183 \).

(b) \( P(X > 2|X > 1) = (1 - F(X > 2))/(1 - P(X > 1)) = e^{-4}/e^{-1/4} = e^{-3.75} = 0.0235 \).

(c) \( P(4 < X < 1.4) = F(1.4) - F(0.4) = e^{-(0.4)^4/4} - e^{-(1.4)^4/4} = 0.6109 \).
Exercise 6 (Bonus). Let $Z = (X - 100)/15$. Then $P(X > 130) = P(Z > 2) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$. Let $X'$ be the random variable $X$, given that $X > 130$. So $X' = X|X > 130$. Similarly let $Z' = Z|Z > 2$. Then $Z' = (X' - 100)/15$. And $EZ' = (EX' - 100)/15$ or equivalently $EX' = 15EZ' + 100$.

The density function of $Z'$ is

$$f_{Z'}(z) = \frac{\phi(z)}{P(Z > 2)} = \frac{e^{-z^2/2}}{\sqrt{2\pi}(0.0228)} = 17.4975e^{-z^2/2}.$$ 

And

$$EZ' = 17.4975 \int_2^\infty z e^{-z^2/2} dz = 17.4975[-e^{-z^2/2}]_2^\infty = 17.4975 e^{-2} = 2.3680$$

Therefore

$$EX' = 15(2.3680) + 100 = 135.52.$$