

MATH 513: LINEAR ALGEBRA ASSIGNMENT 10

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The **Challenging Problems** are due on **Monday, November 26** at noon in class. You do **not** have to hand in the routine problems. On a quiz on Monday, November 26, similar problems may appear. It is optional to hand in the **Very Challenging Problems** (but the same deadline applies). These problems will be very hard. You can earn extra credit with the very challenging problems (but they will be graded more strictly).

READING

Read Chapter 5 on determinants.

ROUTINE PROBLEMS

1. Do Section 17, page 146, problem 1.
2. Do Section 17, page 146, problem 2.
3. Do Section 18, page 149, problem 1.
4. Do Section 19, page 160, problem 1.
5. Do Section 19, page 160, problem 2.

CHALLENGING PROBLEMS

1. (a) Do Section 18, page 150, problem 2.
(b) Do Section 18, page 150, problem 5.
2. (a) Do Section 18, page 150, problem 6.
(b) Do Section 19, page 162, problem 11.
3. Do Section 19, page 161, problem 9.
4. Do Section 19, page 161, problem 10.

VERY CHALLENGING PROBLEMS

1. Let $\mathbf{A} = (\alpha_{i,j})$ be the $n \times n$ matrix with $\alpha_{i,j} = j - i$ if $i \leq j$ and $\alpha_{i,j} = n + j - i$ if $i > j$. For example, for $n = 2$ and $n = 3$ we get the matrices

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

Prove that

$$\det(\mathbf{A}) = \frac{(-1)^{n-1}(n^n - n^{n-1})}{2}.$$