CODING THEORY, MATH 567
PROBLEM SET 3
DUE: MONDAY MARCH 1

(1) §3.1, problem 9.
(2) (a) §3.1, problem 4.
    (b) §3.2, problem 2.
(3) §3.2, problem 4.
(4) Suppose that you have 30 coins. Exactly one of the coin is counterfeit but you do not know which coin. Each coin has equal probability $\frac{1}{30}$ to be counterfeit. You can make three weighings with a balance scale to determine which coin is counterfeit (each weighing has 3 possible outcomes, either the left side is heavier, both sides are in balance or the right side is heavier.) Explain, using Fano's inequality, that it is impossible to determine the counterfeit coin with 100% certainty. Give **some** (positive) lower bound for the decision error probability p_e for this problem.
(5) Prove inequality (3.3.5) on page 100. (Hint: Prove first that $I(X|Y_i) = I(X_i|Y_i)$.)
(6) Do §3.3, Problem 10, part (a),(b), (d).
(7) (a) Do §4.2, problem 5.
    (b) Do §4.2, problem 6.
(8) (Bonus)* Do §4.2, problem 23.
(9) (Bonus) Suppose that we have a (nonsymmetric) binary channel with $P(1|0) = 0$ and $P(0|1) = p$. (So we also have $P(0|0) = 1$ and $P(1|1) = 1 - p$.) What is the capacity of this channel?

Date: Winter 2004.