CODING THEORY, MATH 567
PROBLEM SET 4
DUE: WEDNESDAY APRIL 7

Parts of problems marked with * are more difficult and give bonus points (5 bonus points for each starred part).

(1) Do §5.1, problem 25.

(2) We study the Hamming code $L = \mathcal{H}_q(r)$. This is an $[n, n-r, 3]$ code over the field $\mathbb{F}_q$ where $n = (q^r-1)/(q-1)$. Let us consider first the case $q = 5$ and $r = 2$. So L is a $[6, 4, 3]$ code.
   (a) Write down a 2×6 parity check matrix for $L = \mathcal{H}_5(2)$. Also write down a 4×6 generator matrix for L.
   (b) Show that the weight enumerator $W_{L^\perp}(s)$ for $L^\perp$ is equal to $1 + 24s^5$. (For example, by listing all codewords of $L^\perp$).
   (c) Use the MacWilliams identity to find the weight enumerator $W_L(s)$ of L.

   We go back to the general situation. Let $L = \mathcal{H}_q(r)$ where q and r are arbitrary.
   *(d) Prove that the weight enumerator $W_{L^\perp}(s)$ of the dual code is equal to

$$1 + (q^r - 1)s^{q^{r-1}}$$

   (e) Use (d) to write down a formula for $W_L(s)$, the weight enumerator of L.

(3) Suppose that $n \geq 5$ is a positive integer and assume that a perfect binary 2-error correcting code of length n exists.
   (a) How many codewords would such a perfect binary 2-error correcting code of length n have? Prove that $n^2 + n + 2$ must be a power of 2.
   (b) Assume that C is a perfect 2-error correcting code binary code of length $n = 90$. Assume that C contains the zeroword. Let

$$W_C(s) = A_0 + A_1 s + \cdots + A_{90} s^{90}$$

   be the weight enumerator of C. Explain why $A_0 = 1$ and that $A_1 = A_2 = A_3 = A_4 = 0$ and $A_0 + A_1 + \cdots + A_{90} = 2^{78}$.
   (c) Deduce that $A_5 = 11748$ by counting words of weight 3.

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*(d) Finally, deduce a contradiction by counting words of weight 4 and 5. This shows that no 2-error correcting perfect code of length 90 exists.

(4) (a) Do §6.1, problem 9. (I think I actually mentioned this in class.) More generally, show that every binary linear self-dual code contains the vector with all 1's.

(b) Do §6.1, problem 20.